Boise Extravaganza in Set Theory

Southern Oregon University
Ashland, OR, USA

June 19–20, 2019

Inaccessible Jónsson Cardinals

Shehzad Ahmed
Ohio University
sa066513@ohio.edu

Given an inaccessible Jónsson cardinal λ, a sequence of results due to Shelah tell us that λ must be at least $\lambda \times \omega$-Mahlo. Unfortunately, not much progress has been made on this problem since Sh413 due to the opaque nature of the proofs involved. The goal of this talk is to provide motivation for looking at this problem, and to survey Shelah's results with the hope of making them more accessible. With this in mind, we will also sketch a new proof of the $\lambda \times \omega$ bound based on the work done in the following paper: https://arxiv.org/abs/1901.02417

Applications of the Gandy-Harrington Topology

Shaun Allison
Carnegie Mellon University
sallison@andrew.cmu.edu

We will define the Gandy–Harrington topology of an arbitrary (recursively presented) Polish space and isolate several important properties. Then we will give a recipe which can be used to prove dichotomy theorems concerning Borel orbit equivalence relations. We will briefly sketch a proof of the Harrington-Kechris-Louveau E_0 dichotomy in the special case of orbit equivalence relations, and explain how the proof can be modified to prove two other dichotomies due to Hjorth.

Ramsey theoretic methods in dynamics of topological groups

Dana Bartošová
University of Florida
dbartosova@ufl.edu

In recent decades, certain combinatorial principles of Ramsey theoretic nature have turned out to be equivalent to dynamical properties of topological groups. We will recall both ends of this beautiful connection, discuss recent developments and open problems.

Combinatorics at the First Uncountable Cardinal under AD

William Chan
University of North Texas
William.Chan@unt.edu
Coauthors: Stephen Jackson

This talk will survey some recent results on combinatorics at the first uncountable cardinal under the axiom of determinacy. Continuity results on functions from the power set of the first uncountable cardinal into the set of countable ordinals will be discussed. The talk will mention some uniformization results concerning club subsets of the first uncountable cardinal. Result on definable cardinality and definable combinatorics of subsets of the power set of the first uncountable cardinal assuming AD will be shown as well.

Representation of Functions and Total Antisymmetric Relations in Monadic Third Order Logic

M. Randall Holmes
Boise State University
rholmes@boisestate.edu

We analyze the representation of binary relations in general, and in particular of functions and of total antisymmetric relations, in monadic third order logic, that is, the simple typed theory of sets with three types. We show that there is no general representation of functions or of total antisymmetric relations in this theory. We present partial representations of functions and of total antisymmetric relations which work for large classes of these relations, and show that there is an adequate representation of cardinality in this theory (a result already shown in a somewhat different way by Henrard in unpublished work, but our approach differs from his in providing representations of bijections between sets in a stronger sense). The relation of our work to similar work by Henrard (to whom we are indebted) and Allen Hazen (who arrived at related results independently) is discussed. This work can be understood as part of a program of assessing the capabilities of (relatively) weak logical frameworks: our results are applicable for example, to the framework in David Lewis's Parts of Classes.

The determinacy of some games arising in topology and number theory.

Stephen Jackson
University of North Texas
stephen.jackson@unt.edu
Coauthors: Logan Crone, Lior Fishman, Nathaniel Hiers.

We consider some natural games which arise in topology, analysis, and number theory. In particular we consider the Rothberger game on topological spaces, and the Schmidt game on $\mathbb{R}^d$.

We show that the Rothberger game for T_2 spaces is equivalent to a restricted form of the Menger game which answers a question of Aurichi, Bella, and Dias. For Schmidt games, we investigate their determinacy under AD, and prove a result which extends some work of Becker, Freiling and Martin.

In particular we show these games are determined from AD for $d = 1$, but not in dimension 3 or higher. For $d = 2$ we do not know.

Weak computability in the κ-Turing degrees

Reese Johnston
University of Washington Robinson Center
reesej2@uw.edu
Coauthors: Noah Schweber

We present several set-theoretically natural examples separating weak notions of computable reduction from Turing reductions in the context of κ-recursion for inaccessible cardinals κ. In particular, under sufficiently strong large-cardinal assumptions on κ, we present a natural length-κ sequence of sets that form a strictly descending sequence under many-one reducibility, despite being pairwise Turing-equivalent.

Computable Reducibility of Equivalence Relations and a Jump Operator

Gianni Krakoff
Boise State University
giannikrakoff@u.boisestate.edu

Computable reducibility of equivalence relations is a tool to compare the complexity of equivalence relations on natural numbers. Borel equivalence relation theory motivates questions about computable reducibility. We will define a jump operator on equivalence relations analogous to the Friedman–Stanley Jump and study proprieties of this operation and its iteration. We will then apply this new jump operation by studying its effect on the isomorphism relations of well-founded computable trees as well as c.e. equivalence relations.

What can we say about sets of real numbers closed under Turing equivalence?

Ivan Ongay-Valverde
University of Wisconsin-Madison
ongay@math.wisc.edu

Studying curious properties of subsets of reals has always been an essential part of Set Theory. Following that spirit, I wondered, what can we say about sets of real numbers closed under Turing equivalence?

This question led me to discover that subsets of reals closed under Turing equivalence cannot be Luzin set nor Sierpinski sets; they cannot take all the possible order subtypes of reals but they still have as many as $2^{2^{\aleph_0}}$ order types.

In this talk, I will go over this results and share multiple open questions related to this idea.

100 years of the Borel covering property

Marion Scheepers
Boise State University
mscheepe@boisestate.edu

In a 1919 paper on Lebesgue measure zero sers of real numbers, Borel introduced a covering property. Ten years later in an unrelated investigation F.P. Ramsey published a result now known as Ramsey's Theorem. On account of work over the last two decades or so, Borel's covering property and Ramsey theory are now inextricably linked. In this talk we give a brief report on this confluence of mathematical themes.

Borel reducibility and symmetric models

Assaf Shani
UCLA
assafshani@ucla.edu

We develop a correspondence between the study of Borel equivalence relations induced by closed subgroups of S_∞, and the study of weak choice principles, and apply it to prove a conjecture of Hjorth-Kechris-Louveau (1998). For example, we show that the equivalence relation $\cong^*_{\omega+1,0}$ is strictly below $\cong^*_{\omega+1}$ in Borel reducibility. By results of Hjorth-Kechris-Louveau, $\cong^*_{\omega+1}$ corresponds to $\Sigma^0_{\omega+1}$ actions of S_∞, while $\cong^*_{\omega+1,0}$ corresponds to $\Sigma^0_{\omega+1}$ actions of "well behaved" closed subgroups of S_∞, e.g., abelian groups. For these proofs we analyze the models M_n, $n < \omega$, developed by Monro (1973), and extend his construction past ω, through all countable ordinals. This answers a question of Karagila (2016).

Realizations of countable Borel equivalence relations

Forte Shinko
California Institute of Technology
fshinko@caltech.edu
Coauthors: Alexander Kechris

By a classical result of Feldman and Moore, it is known that every countable Borel equivalence relation can be realized as the orbit equivalence relation of a continuous action of a countable group on a Polish space. However, if we impose further conditions, such as requiring the action to be minimal, then it is no longer clear if such a realization exists. We will detail the progress on characterizing when realizations exist under various conditions, including a complete description in the hyperfinite case. This is joint work with Alexander Kechris.

Formally Verifying Peano Arithmetic

Morgan Sinclaire
Boise State University
morgansinclaire@boisestate.edu
Coauthors: Randall Holmes

Gentzen's consistency proof is a central result in proof theory that demonstrates the consistency of Peano arithmetic (PA) using a technique known as Cut-elimination. The proof can be carried out in the weaker finitist system of primitive recursive arithmetic (PRA), if one extends that system with the principle of transfinite induction over the ordinal ϵ_0. In our work, we have implemented a version of this proof as a computer program in the Coq theorem prover. Consequently, the proof has been computer verified, and many of the interesting constructions in the proof–such as Cantor normal form ordinals and infinitary proof trees–have been built as constructive, finitistic objects.

Selection Principles in Mathematics

Piotr Szewczak
Cardinal Stefan Wyszyński Univeristy in Warsaw, Poland
p.szewczak@wp.pl

The theory of selection principles deals with the possibility of obtaining mathematically significant objects by selecting elements from sequences of sets. The studied properties mainly include covering properties, measure- and category-theoretic properties, and local properties in topological spaces, especially functions spaces. Often, the characterization of a mathematical property using a selection principle is a nontrivial task leading to new insights on the characterized property. I will give an overview of this theory and present some recent results related to this field.