Keeping track of constants.

Recall from class today, that we modeled the motion of an object under the following assumptions:

- Motion occurs along a straight line.
- The mass $m$ of the object is constant.
- Gravitational acceleration is constant: $g \approx 9.81 \text{m/s}^2$.
- The resistive force is proportional to velocity: $F_R = -rv$.

The resulting differential equation is first order in velocity:

$$v' = -g - \frac{rv}{m}$$

where $v = v(t)$ is velocity and $g$, $m$ and $r$ are constants. To solve for $v$, we use the technique of separation of variables.

$$\int \left(g + \frac{r}{m}v\right)^{-1} dv = - \int dt$$

$$\frac{m}{r} \ln \left|g + \frac{r}{m}v\right| = -t + C$$

$C =$ constant of integration

$$\ln \left|g + \frac{r}{m}v\right| = -\frac{r}{m}t + C$$

"new" $C = \frac{r}{m} \times "old" C$

$$\left|g + \frac{r}{m}v\right| = Ce^{-\frac{r}{m}t}$$

"new" $C = e^{"old" C}$: "new" $C > 0$

$$g + \frac{r}{m}v = Ce^{-\frac{r}{m}t}$$

"new" $C = \pm "old" C$

$$v = Ce^{-\frac{r}{m}t} - \frac{mg}{r}$$

"new" $C = \frac{m}{r} \times "old" C$

The exceptional solution (that is, the one not accounted for by the solution technique due to division by zero) occurs when $v = -\frac{mg}{r}$. This corresponds to $C = 0$, and so the general solution is:

$$v = Ce^{-\frac{r}{m}t} - \frac{mg}{r} \quad C \text{ any real number.}$$