Math 333, section 031
Summer 2011 (Ultman)
FINAL EXAM REVIEW

The final exam will be held in class (9:40–11:20 am) on Thursday 28 July 2011. You will be allowed a scientific calculator and both sides of an 8.5" × 11" sheet of paper for handwritten notes in addition to the tables of basic derivatives and integrals and the table of basic Laplace transforms posted on the course website (handwritten additions to the tables are allowed). No other notes or technology will be permitted for this exam.

This exam will be over the entire course. A significant percentage (30 – 75%) will address material covered in class since the second exam, corresponding to sections 8.1, 8.4, 8.5, 9.1–9.3 and 9.5 in the text. This review will address primarily the new material. Review problems will be included for the material preceding the second exam. For a more detailed review of the older material, refer to the reviews for the previous exams. We will discuss/construct a general overview of the course in class on Monday 25 July.

CURRENT MATERIAL

Suggested review problems:

sec. 8.1 # 10, 11, 12, 15

sec. 8.4 # 19, 21, 24–28

Review problems for sections 8.5, 9.1–9.3 and 9.5 are the problems in assignments 11 and 12.
SYSTEMS OF DIFFERENTIAL EQUATIONS

Key Ideas:

- A system of differential equations is a set of differential equations involving the same unknown functions. A solution to a system of differential equations is a set of functions (corresponding to the unknown functions occurring in the equations) which satisfy all equations in the system. A solution to an initial value problem involving a system of differential equations is a solution to the system which also satisfies the initial conditions. Higher-order differential equations can be represented as systems of first order differential equations.

- There are many similarities between the structure of solutions to an \( n \)th-order linear homogeneous differential equation in a single unknown function, and the structure of solutions to a system of \( n \) first-order homogeneous linear differential equations in \( n \) unknown functions. For example, given the \( n \times n \) system \( \vec{x}'(t) = A\vec{x}(t) \):
  - if \( \vec{x}_1(t), \ldots, \vec{x}_m(t) \) are solutions to the system, then so is any linear combination \( \vec{x}(t) = c_1\vec{x}_1(t) + \cdots + c_m\vec{x}_m(t) \) for any choice of constants \( c_1, \ldots, c_m \) (Principle of Superposition);
  - if \( \vec{x}_1(t), \ldots, \vec{x}_n(t) \) are a linearly independent solutions to the system, then any solution \( x(t) \) can be written as a linear combination of the \( \vec{x}_i \)'s; that is, \( x(t) = c_1\vec{x}_1(t) + \cdots + c_n\vec{x}_n(t) \) is the general solution to the system (fundamental solution set/general solution);
  - if \( \vec{x}_1(t), \ldots, \vec{x}_n(t) \) are solutions to the system, then they are linearly independent if and only if their Wronskian is not zero.

So the strategy for finding the solution to an initial value problem with an \( n \times n \) system of first-order homogeneous linear differential equations is to find a fundamental set of solutions (\( n \) linearly independent solutions), which in turn gives the general solution. Then, determine the constants in the general solution so that the solution satisfies the initial conditions. Recall, this is the same strategy used for finding solutions to IVP's with a homogenous linear equation in a single unknown function.
• If an $n \times n$ system of first order homogeneous linear equations has constant coefficients, then linearly independent solutions can be found using the eigenvalues and eigenvectors of the coefficient matrix. The only case in which the solutions so produced do not form a fundamental set of solutions is when there are eigenvalues for which the algebraic multiplicity is greater than the geometric multiplicity (that is, the number of times an eigenvalue is repeated as a root of the characteristic equation exceeds the dimension of its eigenspace). When this happens, there are too few solutions produced. The remedy for this is to make up the remainder of the solutions using generalized eigenvectors.

You are expected to be able to determine whether a given set of function is the solution of a given system of differential equations. Given initial conditions and a general solution, you need to be able to find the particular solution that satisfies the initial conditions. You must be able to convert a higher-order equation into a system of first-order equations.

You need to be able to write a system of linear equations in matrix/vector form. You need to be able to determine whether a given set of solutions to a homogenous linear first-order system is linearly independent, and whether they form a fundamental solution set to the system. You are expected to be able to use the eigenvalue/eigenvector method to compute general solutions (by hand) to two-by-two and three-by-three systems of homogenous linear first-order differential equations with constant coefficients, and should understand how the method extends to larger systems.

Be familiar with the following models described by linear systems with constant coefficients:

- describing the amount of solute in each tank in a multiple tank “mixing problem”;
- describing charge and current in an electrical circuit consisting of multiple loops, where each loop is an LR-, RC-, or RLC-series circuit;
- describing motion of multiple spring/mass systems along a horizontal surface.
SUGGESTED REVIEW PROBLEMS FOR MATERIAL PRECEDING THE SECOND EXAM

There is no way I could provide a comprehensive, exhaustive list of review problems that would address every possible aspect of the course that may appear on the final exam. I have tried to find problems that, taken together, will help you diagnose where your strengths and weaknesses lie and allocate review time accordingly. If you need more review problems, look at the homework for the relevant sections and the suggested review problems for the previous two exams. The “Supplemental exercises” link on the course home page may also be useful if you need to brush up on basic skills.

First order equations:
- sec. 2.2 # 13, 14, 27, 33
- sec. 2.4 # 13, 15, 29, 42
- sec. 2.5 # 1, 4, 5, 7
- sec. 2.6 # 19, 20
- sec. 2.9 # 9, 19, 21, 23, 30, 31
- sec. 3.1 # 5–7, 10, 16, 19
- sec. 3.4 # 13–16

Second order linear equations:
- sec. 4.1 # 15, 25
- sec. 4.2 # 7
- sec. 4.3 # 25, 29, 31, 37, 38
- sec. 4.4 # 16, 18,
- sec. 4.5 # 19, 21, 23, 25, 28, 30
- sec. 4.6 # 2, 13
Laplace transforms:

sec. 5.1 # 3, 28, 29 (for these problems, compute the Laplace transform using the definition)

sec. 5.2 # 7, 11, 17, 29, 37

sec. 5.4 # 32, 33, 37a, 42

sec. 5.5 # 13, 15, 21, 25, 28, 29, 36a

sec. 5.6 # 5, 7

Linear algebra/matrix theory:

sec. 7.1 # 9, 31, 40

sec. 7.3 # 1, 3, 5, 19

sec. 7.6 # 5, 7, 23, 25

sec. 7.7 # 41, 43, 51, 53, 54

sec. 9.1 # 5, 7, 9