1. Let $S$ be the region bounded by $y = 20 - x^2$ and $y = x^2 - 12$.

(a) Sketch the region $S$.

(b) Find the points of intersection of the two graphs.

(c) Partition the region along the $x$-axis and compute the area of a representative rectangle.

(d) Find the area of $S$ using the appropriate integral.
2. Take the portion of the region in problem 1 which is in the first and fourth quadrants.

(10) (a) Using a suitable partition, draw a representative rectangle for that partition. Draw the shell or disk created by revolving that rectangle about the $y$ axis. What is the volume of that shell or disk?

(10) (b) Find the volume when this region (the piece of problem 1 in the first and fourth quadrants) is revolved about the $y$ axis.
(10) 3. Find the average value of \( f(x) = \sqrt{16 - x^2} \) over the interval \([-4, 4]\)
4. A water tank is in the shape of a right circular cone of altitude 10 feet and base radius of 5 feet, with its vertex down.

(a) Draw a picture of the tank and include a horizontal cross section.

(b) What is the volume of this cross section as a function of the distance from the bottom of the tank to the cross section.

(c) If the tank is full, find the work done in pumping all of the water out of the top of the tank. (Water weight 62.5 pounds per cubic foot.)
(20)  5. A wedge is cut from a cylinder of radius 2 by two planes, one perpendicular to the axis of the cylinder and the other at an angle of $\pi/6$ to the first, the planes intersecting on a diameter of the cylinder. Find the volume of the wedge. (Hint: Your slices should be perpendicular to the plane which is perpendicular to the axis of the cylinder.)