This test consists of 4 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

(12) 1. Find the area between the curves $y = \sqrt{x}$ and $y = -\frac{1}{2}x$ over the interval $[0, 4]$. In the process of finding this area, be sure you sketch the region, draw and label the sides of a representative rectangle and form the Riemann sum for the area.

(12) 2. Find the area of the region bounded by the curves $y = x^3 - 4x^2 + 3x$ and $y = 0$. Again, sketch the region and representative rectangles before setting up the integral.
(12) 3. For each positive integer \( n \), let \( A(n) \) be the area between the graph of \( y = \sqrt{x} \) and the graph of \( y = x \).

(a) Find a formula for \( A(n) \) in terms of \( n \). (This means you need to set up an integral and evaluate that integral.)

(b) What is the value of the following limit?
\[
\lim_{n \to \infty} A(n)
\]

(13) 4. Find the volume of the solid that results when the region enclosed by \( y = \sqrt{x}, y = 0 \) and \( x = 9 \) is revolved about the line \( x = 9 \). You need to sketch the region, draw a representative rectangle and a representative piece of the solid as this rectangle is revolved about the line \( x = 9 \).
5. Find the volume of a solid whose base is the region bounded by the curves $y = x$ and $y = x^2$ and whose cross sections perpendicular to the x-axis are squares.

6. Set up an integral (do not evaluate this integral!) whose value is the volume obtained when the region between the graphs of $y = 1/x^3, \quad y = 0, \quad x = 1$ and $x = 2$ is revolved about the line $x = -1$. 
7. A 1600-lb elevator is suspended by a 200-ft cable that weighs 10lb/ft. How much work is required to raise the elevator from the basement to the third floor, a distance of 36 feet?

8. If $f$ is a continuous function, what is the limit as $h \to 0$ of the average value of $f$ on the interval $[x, x + h]$? Be sure to justify your answer. Why is this fact important in calculus?