Please work in groups with no more than four people and complete this worksheet during class. Hand in one worksheet for each group.

1. Use the closed interval method to find the minimum and maximum values of $f(x) = 2x^3 - 15x^2 + 36x$ on $[1, 5]$ and determine where they are.
2. Consider the function \( f(x) = x^4 + 4x + 1 \).

(a) Use the Intermediate Value Theorem to show that this function has at least one real root.

(b) Verify that this function satisfies the hypotheses of Rolle’s Theorem (use your calculator). Then find all numbers \( c \) that satisfy the conclusion of Rolle’s theorem.
(c) Illustrate all values of $c$ found in (b). From the graph of $f(x)$ estimate the number of real roots of $f(x)$.

(d) Assume that there are three roots to the equation $a, b, d$ with $a \leq b \leq d$. Show that the hypotheses of Rolle’s theorem are satisfied on $[a, b]$ and $[b, d]$. 
(e) State the conclusion of Rolle’s theorem on each interval \([a, b]\) and \([b, d]\).

(f) You should notice a problem in your answer in (d). This means that our assumption that there are three roots is incorrect and there are at most two real roots.