These are alleged answers. For each error herein, you get extra-credit points for being the first to report it by e-mail.

1. This is problem (2.4: 3): \( y' = -\frac{2}{x} y + \frac{\cos(x)}{x^2}. \)

We do the **1OLDE** (First-Order Linear differential equation) shtik:

\[
y' + \frac{2}{x} y = \frac{\cos(x)}{x^2}
\]

We can see that an integrating factor can be taken to be \( e^{M(x)} \), where \( M(x) \) is any antiderivative of \( \frac{2}{x} \), the coefficient of \( y \) in the last arrangement of the differential equation:

\[
M(x) = \int \frac{2}{x} \, dx = 2 \ln(|x|) + C_0 = \ln(|x|^2) + C_0 = \ln(x^2) + C_0.
\]

This makes the integrating factor go

\[
e^{M(x)} = e^{\ln(x^2) + C_0} = e^{\ln(x^2)} e^{C_0} = x^2 C_1.
\]

Now we are going to multiply both sides of the differential equation by \( e^{M(x)} \), so \( C_1 \) will just cancel out, so we can just set \( C_1 = 1; \) \( e^{M(x)} = x^2 \).

About now, we realize we could have just eye-balled the integrating factor without all the antiderivative fuss.

Multiplying the differential equation by \( e^{M(x)} \) yields

\[
y' + \frac{2}{x} y = \frac{\cos(x)}{x^2}
\]

\[
x^2 y' + 2xy = \cos(x)
\]

\[
(x^2 y)' = \cos(x)
\]

\[
x^2 y = \sin(x) + C
\]

\[
y = \frac{\sin(x)}{x^2} + \frac{C}{x^2}
\]
2. To solve the initial-value problem \( y' = \frac{2}{x} y + \frac{\cos(x)}{x^2} \) with \( y(\pi/6) = 36/\pi^2 \), we use the initial condition to evaluate the constant \( C \) in our final answer to question 1:

\[
\frac{36}{\pi^2} = \frac{\sin(\pi/6)}{(\pi/6)^2} + \frac{C}{(\pi/6)^2} \\
\frac{1}{(\pi/6)^2} = \frac{\sin(\pi/6)}{(\pi/6)^2} + \frac{C}{(\pi/6)^2} \\
1 = \sin(\pi/6) + C \\
1 = \frac{1}{2} + C \\
C = \frac{1}{2}
\]

so that the initial-value problem’s solution is

\[
y = \frac{\sin(x) + \frac{1}{2}}{x^2}
\]

or

\[
y = \frac{2 \sin(x) + 1}{2x^2}.
\]