1 Finite Differences (Approximating derivatives of functions) – Chapter 6.1

- How are finite difference (FD) formulas derived? Give an example.

- When is it appropriate to use a centered FD formula? How about a one-sided formula?

- FD stencil notation:

- Know the simple method for testing the order of accuracy of a FD formula:

- Compact (or implicit) FD formulas:
2 Numerical methods for ODEs (Initial value problems) – Chapter 7

2.1 Theory

• Changing a higher order ODE to a first order system:

• Definition of a Lipshitz continuous function:

• Existence and uniqueness theorem for an IVP:

• Formal solution of an IVP (integral form):

• Definition of a well-posed IVP:

• Definition of convergence for an ODE method:

• Definition of the order of accuracy an ODE method:
2.2 Linear multistep methods

- Stencil notation:

- Generating polynomial notation:

- Know the simple method for testing the order of accuracy of a LMS method:

- Definition of stability

- Theorem relating stability of a LMS method to the roots of its characteristic polynomial:

- Definition of consistency:

- Theorem relating consistency and order of accuracy:
• Dahlquist equivalence theorem:

• First Dahlquist stability barrier and its application:

• How are Adams-Bashforth (AB) and Adams-Moulton (AM) methods derived? What is the main difference between an AB and AM method?

• How are backward differentiation formulae (BDF) derived? What is the main difference between Adams methods and BDF methods?

• What is the idea behind predictor-corrector methods?

2.3 Runge-Kutta methods
• How are they derived (just the basics)?

• How do they differ from linear multistep methods?

• Butcher diagram notation.

• Implicit vs. explicit.
2.4 Stability domains

- Definition of a stability domain:

- What is the idealized problem we use to determine a methods stability domain? What does stability depend on?

- General idea for computing stability domains?

- How are stability domains used?

2.5 Stiff IVPs

- General themes defining a stiff IVP

- Stiffness ratio:

- Existence and uniqueness theorem for an IVP:
• Definition of $A$-stability? Why is $A$-stability important?

• What methods are $A$-stable?

• Definition of $A(\alpha)$-stability?