Math 326 **Midterm Exam 2 Practice** **Midterm: April 17, 2008**

Administrative Details
Scientific calculators: No lap tops One side of standard 8.5 × 11 paper with notes Show all work. Cirlce/box the final answer.

<u>Problems</u>: Solve the following problems showing all work.

1. Evaluate the following integrals

   (a) $\int_C \frac{z}{\overline{z}^2} dz$, where C is one eighth of the circle $|z| = 2$ from $z = 0$ to $z = 2e^{i\pi/4}$.

   (b) $\int_C \frac{1}{z^{1/2}} dz$, where C is any contour from $z = -1$ to $z = 1$ that, except for the end points, lies below the real axis and the integrand is the branch $z^{1/2} = \sqrt{r}e^{i\theta/2}$ $(r > 0,\ \pi/2 < \theta < 5\pi/2)$.

   (c) $\int_C e^{\overline{z}} dz$, where C is the segment $-\pi \leq y \leq \pi$ on the imaginary axis.

   (d) $\int_C z^{1/4} dz$, where C is any contour from $z = -i$ to $z = i$ that, except for the end points, lies to the right of the imaginary axis and the integrand is the branch $z^{1/4} = r^{1/4}e^{i\theta/4}$ $(r > 0,\ \pi < \theta < 3\pi)$.

2. Find the maximum value of $u(x, y) = x\cos(y)\sinh(x) - y\cosh(x)\sin(y)$ on the square R: $0 \leq x \leq \pi/2$, $0 \leq y \leq \pi/2$, and specify where the maximum occurs. Justify your answer.

3. Compute the value of the following integrals over the simple closed contour C: $|z| = 2$:

   (a) $\int_C \frac{e^{i\pi z}}{(z-1)(z^3-9)} dz$.

   (b) $\int_C \frac{z}{(z+3i)(z^2-2iz-1)} dz$.

   (c) $\int_C \frac{\sin z}{z} dz$.

   (d) $\int_C \frac{\sinh z}{z^4} dz$.

4. Determine the Taylor series expansion for each of the following functions about the specified point and draw the region of convergence:

$$\text{(a) } f(z) = \cos(z + \pi/2), \text{ about } z = 0$$

$$\text{(b) } f(z) = \frac{1}{1-z}, \text{ about } z = 1/2$$

5. Determine the Taylor series expansion for the following function about the specified point and give the region of convergence:

$$f(z) = \sinh z, \text{ about } z = \pi i$$

6. Determine the Laurent series expansion that is valid in the specified domain for the function

$$f(z) = \frac{e^z}{z - i\pi}, \text{ for } 0 < |z - i\pi| < \infty$$

7. Determine the Laurent series expansion for each of the following functions that is valid in the specified domain:

$$\text{(a) } f(z) = (z-1)e^{\frac{1}{z-1}}, \text{ for } 0 < |z-1| < \infty$$

$$\text{(b) } f(z) = \frac{1}{z(2+z)}, \text{ for } 2 < |z| < \infty$$

8. Given the Laurent series expansion for $f(z) = 1/\sin z$ about $z = 0$ that is convergent for $0 < |z| < \pi$

$$\frac{1}{\sin z} = \frac{1}{z} + \frac{z}{6} + \frac{7z^3}{360} + \frac{31z^5}{15120} + \cdots,$$

determine the values of the integrals

$$\text{(a) } \int_C \frac{1}{\sin z} dz \; .$$

$$\text{(b) } \int_C \frac{1}{z^2 \sinh z} dz \; ,$$

where C is the positively oriented contour given by $|z| = 2$ (i.e. circle centered at the origin of radius 2).