

**Administrative Details**

Scientific calculators: No lap tops  
One side of standard  $8.5 \times 11$  paper with notes

**Problems:** Solve the following problems showing all work.

1. Using the result that

$$\sin \theta = \frac{e^{i\theta} - e^{-i\theta}}{2i},$$

derive the trigonometric identity

$$\sin^2 \frac{\theta}{2} = \frac{1}{2}[1 - \cos \theta].$$

2. (a) Put the following complex numbers in polar form, and state their principal argument  
Arg  $z$ :

$$(i) z = 2 + 2i^3 \qquad (ii) z = \frac{1}{(1+i)^2}$$

- (b) Solve for all roots of the following equation

$$z^4 + 3iz^2 - 2 = 0$$

3. Sketch the region onto which the rectangle,  $R$ ,  $0 \leq x \leq 1$ ,  $0 \leq y < \pi$ , in the  $z$ -plane ( $z = x + iy$ ) is mapped by the transformation  $w = e^{2z}$ .
4. Is the function  $f(z) = e^{\bar{z}} + e^z$  (i) continuous, (ii) analytic? Show/explain why or why not.

5. Show that

$$u(r, \theta) = \frac{1}{2\pi r} \sin \theta$$

is harmonic in some domain and find a harmonic conjugate  $v(r, \theta)$ .

6. Determine the value of  $(-i)^{1+2i}$  when

- (a) The principal branch of the log is used (i.e.  $\log = \text{Log}$ ).  
(b) The branch  $\log z = \ln r + i\theta$  for ( $r > 0, 21\pi < \theta < 23\pi$ ).

7. Determine all values of

- (a)  $w = \tan^{-1}(2 + i)$ .  
(b)  $w = \tanh^{-1}(1 + 2i)$ .