Math 326 **Final Exam Practice A** **May 13, 2008**
Grady Wright **10:30–12:30pm**

Administrative Details

Calculators (not mini-computers) are permitted
Both sides of standard 8.5×11 paper with notes
Show all work

Problems:

1. (a) Put the following complex number in polar form ($z = re^{i\theta}$, $r > 0$, $-\pi \leq \theta \leq \pi$):

$$z = \frac{i}{\sqrt{3} + i^3}$$

(b) Put the following complex number in Cartesian form ($z = x + iy$):

$$z = \frac{1}{ie^{i5\pi/2}}$$

(c) Solve for all roots of the following equation (you may leave the answer in either exponential or Cartesian form):

$$z^5 - 2\sqrt{3}z^3 + 4z = 0$$

2. Use contour integration to compute the value of the following improper integral:

$$\int_0^\infty \frac{\cos ax}{x^4 + 1} dx \quad (a \text{ is a real number})$$

3. Compute the inverse Laplace transform of the following function

$$\widehat{F}(s) = \frac{s}{(s-a)(s^2 + a^2)} \quad (a > 0)$$

4. For part (a) and (b) below

   (i) Determine all of the singular points in the finite complex plane;
   (ii) Determine the type of singular point;
   (iii) Determine the residues at each singular point that is a pole.

   (a) $f(z) = z \cosh \dfrac{1}{2z}$

   (b) $f(z) = \dfrac{\sin 2z}{z^2}$

5. Sketch the region onto which the perimeter of the unit circle ($r = 1$, $-\pi \leq \theta \leq \pi$) in the z-plane ($z = re^{i\theta}$) is mapped by the transformation $w = z + \frac{i}{z}$.

6. (a) Show that the function

$$u(x, y) = \sinh x \sin y$$

is harmonic in some domain and find a harmonic conjugate $v(x, y)$.

(b) Determine the maximum value of $u(x, y)$ on the square R: $0 \leq x \leq \pi/2$, $0 \leq y \leq \pi/2$, and specify where the maximum occurs. Justify your answer.

7. (a) Determine the Taylor series of the following function in the given region

$$f(z) = \frac{e^z - e}{z - 1}, \quad |z - 1| < \infty$$

(b) Determine the Laurent series of the following function in the given region

$$f(z) = \frac{z}{a^2 - z^2}, \quad |z| > a \ \ (a > 0)$$

8. Evaluate the following integrals

(a)

$$\int_C z^{1/5} dz \ ,$$

where C is any contour from $z = (-1 - i)/\sqrt{2}$ to $z = i$ that does not pass through the second quadrant (i.e. $x \leq 0$ and $y \geq 0$), and the integrand is the branch $z^{1/5} = r^{1/5} e^{i\theta/5}$ $(r > 0, \ \pi < \theta < 3\pi)$.

(b)

$$\int_C \sin(\overline{z}) dz \ ,$$

where C is the straight line segment from $z = 0$ to $z = 1 + i$.

9. Find all the roots of the equation

$$\tanh w = \sqrt{2} + i$$