Problems:

1. (a) Put the following complex number in polar form \( z = r e^{i\theta}, \ r > 0, \ -\pi \leq \theta \leq \pi \):
   \[
   z = \frac{i}{\sqrt{3} + i^3}
   \]
   (b) Put the following complex number in Cartesian form \( z = x + iy \):
   \[
   z = \frac{1}{ie^{i5\pi/2}}
   \]
   (c) Solve for all roots of the following equation (you may leave the answer in either exponential or Cartesian form):
   \[
   z^5 - 2\sqrt{3}z^3 + 4z = 0
   \]

2. Use contour integration to compute the value of the following improper integral:
   \[
   \int_0^\infty \frac{\cos ax}{x^4 + 1} \, dx \quad (a \text{ is a real number})
   \]

3. Compute the inverse Laplace transform of the following function
   \[
   \mathcal{F}(s) = \frac{s}{(s-a)(s^2 + a^2)} \quad (a > 0)
   \]

4. For part (a) and (b) below
   (i) Determine all of the singular points in the finite complex plane;
   (ii) Determine the type of singular point;
   (iii) Determine the residues at each singular point that is a pole.
   (a) \( f(z) = z \cosh \frac{1}{2z} \)
   (b) \( f(z) = \frac{\sin 2z}{z^2} \)

5. Sketch the region onto which the perimeter of the unit circle \((r = 1, \ -\pi \leq \theta \leq \pi)\) in the \(z\)-plane \((z = re^{i\theta})\) is mapped by the transformation \(w = z + \frac{i}{z}\).

6. (a) Show that the function
   \[
   u(x, y) = \sinh x \sin y
   \]
   is harmonic in some domain and find a harmonic conjugate \(v(x, y)\).
   (b) Determine the maximum value of \(u(x, y)\) on the square \(R: \ 0 \leq x \leq \pi/2, \ 0 \leq y \leq \pi/2\), and specify where the maximum occurs. Justify your answer.
7. (a) Determine the Taylor series of the following function in the given region
\[ f(z) = \frac{e^z - e}{z - 1}, \quad |z - 1| < \infty \]

(b) Determine the Laurent series of the following function in the given region
\[ f(z) = \frac{z}{a^2 - z^2}, \quad |z| > a \quad (a > 0) \]

8. Evaluate the following integrals
(a) \[ \int_C z^{1/5} \, dz, \]
where \( C \) is any contour from \( z = (-1 - i)/\sqrt{2} \) to \( z = i \) that does not pass through the second quadrant (i.e. \( x \leq 0 \) and \( y \geq 0 \)), and the integrand is the branch \( z^{1/5} = r^{1/5} e^{\theta/5} \) \( (r > 0, \pi < \theta < 3\pi) \).
(b) \[ \int_C \sin(\pi) \, dz, \]
where \( C \) is the straight line segment from \( z = 0 \) to \( z = 1 + i \).

9. Find all the roots of the equation
\[ \tanh w = \sqrt{2} + i \]