**Math 297** **Homework 3** **Due Oct. 9, 2008**
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1. As discussed in class, Lagrange's interpolation formula can be rearranged into the magnificient *barycentric formula* (or more specifically the second (true) form of the barycentric formula):

$$p_n(x) = \frac{\sum\limits_{j=0}^{n} \dfrac{w_j}{x - x_j} f_j}{\sum\limits_{j=0}^{n} \dfrac{w_j}{x - x_j}}\ , \tag{1}$$

where

$$w_j = \frac{1}{\prod\limits_{\substack{i=0\\ i\neq j}}^{n} (x_j - x_i)}\ , j = 0, 1, \ldots, n\ .$$

Here $x_j$ are the given nodes and $f_j$ the corresponding function values.

(a) Write a MATLAB function for computing the barycentric weights $w_j$ in the above expression. Your function should take as input a vector containing the nodes $x_j$ and output the weights $w_j$. Call your function `baryfit`. The function declaration should be something like:

```
function w = baryfit(x)
```

Write another function for evaluating the barycentric interpolant $p_n(x)$. This function should take as input a vector containing the nodes $x_j$, a vector containing the corresponding barycentric weights $w_j$ (generated from your `baryfit` function), a vector containing the corresponding function values $f_j$, and the location (or a vector of locations) of where the interpolant should be evaluated. The output of the function should be the value of the interpolating polynomial at all the evaluation points. Call this function `baryeval`. The function declaration should be something like:

```
function p = baryval(x,w,f,xi)
```

Both of these functions should avoid unnecessary loops and FLOPs. Turn in a print out of both functions.

MATLAB has a function `prod` that can be used to compute the product of all entries in a vector.

(b) The three following node sets are known as equispaced, Chebyshev, and Legendre, respectively:

(i) $x_j = -1 + \frac{2j}{8},\ j = 0, 1, \ldots, 8$
(ii) $x_j = -\cos\frac{j\pi}{8},\ j = 0, 1, \ldots, 8$
(iii) $\mp 0.96816023950763$ $\mp 0.83603110732664$ $\mp 0.61337143270059$ $\mp 0.32425342340381$ $0$

For each of these three node sets, use your `baryval` function to evaluate the $8^{\text{th}}$ degree polynomial interpolant of the function $f(x) = |x|$ at 201 equally spaced points between $[-1, 1]$ (i.e. at the points $t_j = -1 + \frac{2j}{200},\ j = 0, 1, \ldots, 200$). Plot the error, $E(x) = p_8(x) - |x|$, in the polynomial interpolant at these evaluation points for each of the three node sets. Which node set seems to produce the best result? What criteria did you use to determine what 'best' means?

(c) Sample the function $f(x) = |x|+x/2-x^2$, at the Chebyshev node points $x_j = -\cos\frac{j\pi}{100}$, $j = 0, 1, \ldots, 100$. Compute and evaluate the interpolant at 1001 equally spaced points over $[-1, 1]$ using your `baryfit` and `baryval` functions and the NCM book's `polyinterp` function. Compare the time it takes both techniques to perform this task.

2. Union Pacific has decided to reopen the Boise Train Depot for rail travel. They have contracted your scientific computing consulting company to design a new switching path between two of the rail lines (Track A and B) that enter the Depot. The requirements for the path are that it pass through the points $(0, 0)$, $(2, 1)$, and $(4, 2)$ (see Figure below). Furthermore, the path should be tangent to the line $y = 0$ at $(0, 0)$, tangent to the line $y = 2$ at $(4, 2)$, *and* have a slope of $3/2$ at $x = 2$.


a Since you are given both the function and derivative values at each of the node points, you decide to use a piecewise cubic Hermite polynomial to model the switching path. Analytically compute the piecewise polynomial for the intervals $[-1, 0]$,$[0, 2]$,$[2, 4]$,$[4, 5]$.

b Make a nice plot the solution from part (a) for Union Pacific.

3. NCM, problem 3.2

4. NCM, problem 3.4

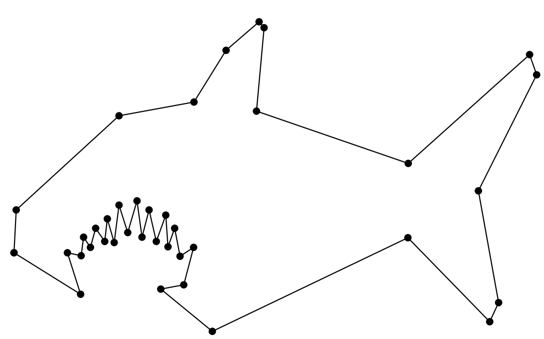
5. Closed curve interpolation.

(a) For the first part of this problem do problem 3.13 (a) from the NCM book.

(b) Download the `shark.mat` file from the course web page to your working MATLAB directory. Execute the following commands:

```
load shark;
plot(x,y,'k.-','MarkerSize',10);
```

This should load into MATLAB two vectors `x` and `y` that represent the cooordinates of the cartoon shark shown below.



(c) For the shark data in part (b), generate a smooth parametric curve for drawing the shark using the `pchiptx`, `splinetx`, and `baryfit` (from problem 1) functions. Make a plot of the shark for each of these functions.

(d) Repeat part (c) for the "periodic" versions of `pchiptx` and `splinetx` that you created in part (a).

(e) Which representation of the shark to you like best and why?