Math 175-004 **Midterm 1 Review** **Feb. 14, 2008**

General Points:

- There is just one fundamental way to prepare for an exam. Understand the material!
- You'll answer questions on the exam itself. All you need to bring is a writing utensil.
- When you receive the exam, **relax** and proceed deliberately. If you don't know how to do a problem, skip it and return to it later. Accuracy is paramount, speed is useless!
- Check your answers.
- During the exam, **all books, notes, and electronic devices must be out of sight.**

Exam topics: Chapter 6.

6.1: Volumes by disks and washers

1. Computing volumes of a solid with known integrable cross-sectional area:

$$V = \int_a^b A(x)dx$$

2. Computing volumes formed by revolving a plane region about the x- or y-axis using the *disk method*:

$$V = \int_a^b \pi[R(x)]^2 dx$$

$$V = \int_c^d \pi[R(y)]^2 dy$$

3. Computing volumes formed by revolving a plane region about the x- or y-axis using the *washer method*:

$$V = \int_a^b \pi([R(x)]^2 - [r(x)]^2)dx$$

$$V = \int_c^d \pi([R(y)]^2 - [r(y)]^2)dy$$

6.2: Volumes by cylindrical shells

1. Computing volumes formed by revolving a plane region about the x- or y-axis using the *cylindrical shell method*:

$$V = \int_a^b 2\pi \begin{pmatrix}\text{shell}\\ \text{radius}\end{pmatrix} \begin{pmatrix}\text{shell}\\ \text{height}\end{pmatrix} dx$$

2. See summary on p. 406.

6.3: Lengths of plane curves

1. Computing the length of a plane curve that is defined parametrically, $(x, y) = (f(t), g(t))$, $a \le t \le b$:

$$L = \int_a^b \sqrt{[f'(t)]^2 + [g'(t)]^2} dt$$

2. Computing the length of a plane curve that is defined by an equation $y = f(x)$ or $x = g(y)$:

$$L = \int_a^b \sqrt{1 + [f'(x)]^2} dx$$

$$L = \int_c^d \sqrt{1 + [g'(y)]^2} dy$$

6.4: Areas of surfaces of revolution

1. If the surface is generated by revolving the function $f(x) \geq 0$, $a \leq x \leq b$ about the x-axis then

$$S = \int_a^b 2\pi f(x) \sqrt{1 + [f'(x)]^2} dx$$

2. If the surface is generated by revolving the function $g(y) \geq 0$, $c \leq y \leq d$ about the y-axis then

$$S = \int_c^d 2\pi g(y) \sqrt{1 + [g'(y)]^2} dy$$

3. If the surface are is generated by revolving the parametrically defined $(x, y) = (f(t), g(t))$, $a \leq t \leq b$ about the x-axis then

$$S = \int_a^b 2\pi g(t) \sqrt{[f'(t)]^2 + [g'(t)]^2} dt,$$

or about the y-axis then

$$S = \int_a^b 2\pi f(t) \sqrt{[f'(t)]^2 + [g'(t)]^2} dt,$$

6.5: Exponential change

1. Know the solution to the differential equation modeling exponential change:

$$\frac{dy}{dt} = ky, \quad y(0) = y_0, \quad t > 0$$

2. Applications of this equation for population growth, radioactive decay, and heat transfer (i.e. Newton's law of cooling).

6.6: Work

1. Definition of work in the case of a variable force in the direction of motion along the x-axis from $x = a$ to $x = b$:

$$W = \int_a^b F(x) dx$$

2. Hooke's law and applications
3. Pumping liquids from containers

6.7: Moments and center of mass

1. Computing the center of mass $(\overline{x}, \overline{y})$ of a thin plate with a density that varies continuously in one of the variables:

$$\overline{x} = \frac{M_y}{M} = \frac{\int \tilde{x} dm}{\int dm}$$

$$\overline{y} = \frac{M_x}{M} = \frac{\int \tilde{y} dm}{\int dm}$$