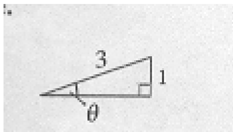
Math 144 Analytic Trigonometry Name ________________________

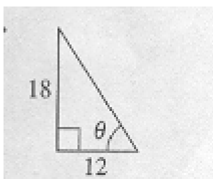
Problem Set #1

Chapter 6 KEY

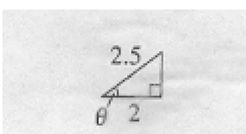
Solve the following triangles. **(2 points each blank)**

1. 

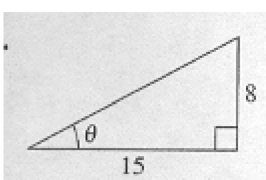
$\theta =$ $\underline{19.46° = 0.3398\,rad}$ $\sin\theta = \dfrac{1}{3} \approx 0.3333$ $\sin^{-1}\left(\dfrac{1}{3}\right) = \boxed{19.46° = 0.3398\,rad}$

2. 

$\theta =$ $\underline{56.31° = 0.9828\,rad}$ $\tan\theta = \dfrac{18}{12} = 1.5$ $\tan^{-1}\left(\dfrac{18}{12}\right) = \boxed{56.31° = 0.9828\,rad}$

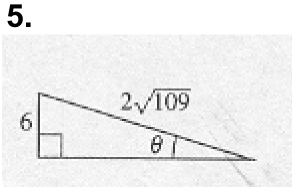
3. 

$\theta =$ $\underline{36.87° = 0.6435\,rad}$ $\cos\theta = \dfrac{2}{2.5} = 1.5$ $\cos^{-1}\left(\dfrac{2}{2.5}\right) = \boxed{36.87° = 0.6435\,rad}$

4. 

$\theta =$ $\underline{28.07° = 0.4900\,rad}$ $\tan\theta = \dfrac{8}{15} = 5.\overline{33}$ $\tan^{-1}\left(\dfrac{8}{15}\right) = \boxed{28.07° = 0.4900\,rad}$

5.



$\theta =$ $\underline{16.70^\circ \approx 0.2915\,rad}$ $\quad \sin\theta = \frac{6}{2\sqrt{109}} \approx 0.2873 \quad \sin^{-1}\left(\frac{6}{2\sqrt{109}}\right) \approx \boxed{16.70^\circ \approx 0.2915\,rad}$

Solve the following triangles. **(1 point each blank)**

6. $a = 5$ $\angle A =$ ________

$b = 9$ $\angle B =$ ________

$c = 12$ $\angle C =$ ________

$Area =$ ________

$$\angle A = \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right) = \cos^{-1}\left(\frac{5^2 - 9^2 - 12^2}{-2 \cdot 9 \cdot 12}\right) = \cos^{-1}\left(\frac{-200}{-216}\right) = \boxed{22.19^\circ = 0.387\ rad}$$

$$\angle B = \cos^{-1}\left(\frac{b^2 - a^2 - c^2}{-2ac}\right) = \cos^{-1}\left(\frac{9^2 - 5^2 - 12^2}{-2 \cdot 5 \cdot 12}\right) = \cos^{-1}\left(\frac{-88}{-120}\right) = \boxed{42.83^\circ = 0.748\ rad}$$

$$\angle C = \cos^{-1}\left(\frac{c^2 - a^2 - b^2}{-2ab}\right) = \cos^{-1}\left(\frac{12^2 - 5^2 - 9^2}{-2 \cdot 5 \cdot 9}\right) = \cos^{-1}\left(\frac{38}{-90}\right) = \boxed{114.97^\circ = 2.01\,rad}$$

$$s = \left(\frac{5 + 9 + 12}{2}\right) = 13 \quad Area = \sqrt{(13)(13-5)(13-9)(13-12)} = \sqrt{13 \cdot 8 \cdot 4 \cdot 1} = \boxed{\sqrt{416} \approx 20.40}$$

7. $a = 14$ $\angle A =$ ________

$b = 18$ $\angle B =$ ________

$c = 7$ $\angle C =$ ________

$Area =$ ________

$$\angle A = \cos^{-1}\left(\frac{a^2 - b^2 - c^2}{-2bc}\right) = \cos^{-1}\left(\frac{14^2 - 18^2 - 7^2}{-2 \cdot 18 \cdot 7}\right) = \cos^{-1}\left(\frac{-177}{-252}\right) = \boxed{45.38^\circ = 0.79\ rad}$$

$$\angle B = \cos^{-1}\left(\frac{b^2 - a^2 - c^2}{-2ac}\right) = \cos^{-1}\left(\frac{18^2 - 14^2 - 7^2}{-2 \cdot 14 \cdot 7}\right) = \cos^{-1}\left(\frac{79}{-196}\right) = \boxed{113.78^\circ = 1.99\ rad}$$

$$\angle C = \cos^{-1}\left(\frac{c^2 - a^2 - b^2}{-2ab}\right) = \cos^{-1}\left(\frac{7^2 - 14^2 - 18^2}{-2 \cdot 14 \cdot 18}\right) = \cos^{-1}\left(\frac{-471}{-504}\right) = \boxed{20.85^\circ = 0.364\ rad}$$

$$s = \left(\frac{14 + 18 + 7}{2}\right) = 19.5 \quad Area = \sqrt{(19.5)(19.5-14)(19.5-18)(19.5-7)} =$$

$$= \sqrt{19.5 \cdot 5.5 \cdot 1.5 \cdot 12.5} = \boxed{\sqrt{2010.9375} \approx 44.84}$$

8. $\angle A = 85°$ $\quad c =$ ________

$b = 24$ $\quad \angle B =$ ________ **Triangle not possible**

$a = 15$ $\quad \angle C =$ ________

$Area =$ ________

9. $\angle A = 42°$ $\quad c =$ ________

$b = 13$ $\quad \angle B =$ ________

$a = 22$ $\quad \angle C =$ ________

$Area =$ ________

$$\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \sin B = \frac{b \cdot \sin A}{a} \Rightarrow$$

$$\angle B = \sin^{-1}\left(\frac{13 \cdot \sin(42°)}{22}\right) = \sin^{-1}\left(\frac{(13)(0.66913)}{(22)}\right) \approx \boxed{23.29° \approx -0.572\ rad}$$

$$\angle C = 180 - \measuredangle A - \measuredangle B \Rightarrow \angle C = 180 - 42 - 23.29 = \boxed{114.71° \approx 0.637\pi\ rad}$$

$$c^2 = a^2 + b^2 - 2ab\cos C \Rightarrow c = \sqrt{a^2 + b^2 - 2ab\cos C} \Rightarrow c = \sqrt{22^2 + 13^2 - 2 \cdot 22 \cdot 13 \cdot \cos(114.71°)} =$$

$$c = \sqrt{484 + 169 - (572) \cdot (-0.41803)} = \sqrt{892.1107} \approx \boxed{29.868}$$

$$s = \left(\frac{22 + 13 + 29.868}{2}\right) = 32.434 \quad Area = \sqrt{(32.434)(32.434 - 22)(32.434 - 13)(32.434 - 29.868)} =$$

$$= \sqrt{(32.434)(10.434)(19.434)(2.566)} = \boxed{\sqrt{16,876.03} \approx 129.907}$$

10. $\angle A = 20.5°$ $\quad c =$ ________

$b = 31$ $\quad \angle B =$ ________

$a = 12$ $\quad \angle C =$ ________ This is an ambiguous case.

$$\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \sin B = \frac{b \cdot \sin A}{a} \Rightarrow$$

$$\angle B = \sin^{-1}\left(\frac{31 \cdot \sin(20.5°)}{12}\right) = \sin^{-1}\left(\frac{(31)(0.3502)}{(12)}\right) \approx \boxed{64.79° \approx 1.13\ rad}$$

$$\angle C = 180 - \measuredangle A - \measuredangle B \Rightarrow \angle C = 180 - 20.5 - 64.79 = \boxed{94.71° \approx 1.65\ rad}$$

$$c^2 = a^2 + b^2 - 2ab\cos C \Rightarrow c = \sqrt{a^2 + b^2 - 2ab\cos C} \Rightarrow c = \sqrt{12^2 + 31^2 - 2 \cdot 12 \cdot 31 \cdot \cos(94.71°)} =$$

$$c = \sqrt{144 + 961 - (744) \cdot (-0.0821)} = \sqrt{1166.0824} \approx \boxed{34.15}$$

$$s = \left(\frac{22 + 13 + 34.15}{2}\right) \approx 34.575 \quad Area = \sqrt{(34.575)(34.575 - 22)(34.575 - 13)(34.575 - 10.55)} =$$

$$= \sqrt{(34.575)(12.575)(21.575)(24.025)} = \boxed{\sqrt{225{,}363.9174} \approx 474.73}$$

OR

$$\frac{\sin A}{a} = \frac{\sin B}{b} \Rightarrow \sin B = \frac{b \cdot \sin A}{a} \Rightarrow$$

$$\angle B = \sin^{-1}\left(\frac{31 \cdot \sin(20.5°)}{12}\right) = \sin^{-1}\left(\frac{(31)(0.3502)}{(12)}\right) \approx \boxed{115.22° \approx 0.6401\pi\ rad}$$

$$\angle C = 180 - \measuredangle A - \measuredangle B \Rightarrow \angle C = 180 - 20.5 - 115.22 = \boxed{44.28° \approx 0.246\pi\ rad}$$

$$c^2 = a^2 + b^2 - 2ab\cos C \Rightarrow c = \sqrt{a^2 + b^2 - 2ab\cos C} \Rightarrow c = \sqrt{12^2 + 31^2 - 2 \cdot 12 \cdot 31 \cdot \cos(94.71°)} =$$

$$c = \sqrt{144 + 961 - (744) \cdot (0.7123)} = \sqrt{575.0488} \approx \boxed{23.98}$$

$$s = \left(\frac{22 + 13 + 10.55}{2}\right) \approx 22.78 \quad Area = \sqrt{(22.78)(22.78 - 22)(22.78 - 13)(22.78 - 10.55)} =$$

$$= \sqrt{(22.78)(0.78)(9.78)(12.23)} = \boxed{\sqrt{2{,}125.2677} \approx 46.10}$$

Evaluate without your calculator. (1 point each blank)

	$\sin\theta$	$\cos\theta$	$\tan\theta$
11. $\theta = 315° = 45°$ in 4th Quad	$-\frac{\sqrt{2}}{2} \approx -0.7072$	$\frac{\sqrt{2}}{2} \approx 0.7072$	1
12. $\theta = -750° = 30°$ in 4th Quad	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2} \approx 0.8660$	$\frac{\sqrt{3}}{3} \approx 0.5774$

13. $\theta = -330° = 30°$ in 4th Quad	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2} \approx 0.8660$	$\frac{\sqrt{3}}{1} \approx 1.7321$
14. $\theta = 660° = 60°$ in 4th Quad	$-\frac{\sqrt{3}}{2} \approx -0.8660$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{1} \approx -1.7321$
15. $\theta = \frac{8\pi}{3}$ rad $= \frac{\pi}{3}$ rad in 2nd Quad	$\frac{\sqrt{3}}{2} \approx 0.8660$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{1} \approx -1.7321$
16. $\theta = -\frac{7\pi}{6}$ rad $= \frac{\pi}{6}$ rad in 3rd Quad	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2} \approx -0.8660$	$-\frac{\sqrt{3}}{3} \approx 0.5774$
17. $\theta = \frac{7\pi}{4}$ rad $= \frac{\pi}{4}$ rad in 4th Quad	$-\frac{\sqrt{2}}{2} \approx -0.7071$	$\frac{\sqrt{2}}{2} \approx 0.7071$	-1
18. $\theta = -\frac{13\pi}{6}$ rad $= \frac{\pi}{6}$ rad in 1st Quad	$\frac{1}{2}$	$\frac{\sqrt{3}}{2} \approx 0.8660$	$\frac{\sqrt{3}}{3} \approx 0.5774$

Use your calculator (round to 2 decimal places) to find TWO different values of θ.

(1 point each blank)

19. $\sin\theta = 0.6428$ $\theta \approx$	40°	140°	*either* $+360n$
20. $\tan\theta = 1.7321$ $\theta \approx$	60°	240°	*either* $+360n$
21. $\sec\theta = 3.8637$ $\theta \approx$	75°	295°	*either* $+360n$
22. $\sin\theta = -0.7071$ $\theta \approx$	135°	315°	*either* $+360n$
24. $\cos\theta = 0.6428$ $\theta \approx$	50°	310°	*either* $+360n$
25. $\tan\theta = -0.2679$ $\theta \approx$	165°	345°	*either* $+360n$

Using trig identities and / or your calculator to solve the equation for x, $0 \le x \le 2\pi$

(3 points each)

26. $(3)(\sec x) + 4 = 10$

$(3)(\sec x) + 4 = 10$

$(3)(\sec x) = 6$

$\sec x = 2$

$\frac{1}{\cos x} = \frac{2}{1}$

$\cos x = \frac{1}{2}$ $\quad \cos^{-1} \cos x = \cos^{-1}\left(\frac{1}{2}\right)$ $\quad \boxed{x = \frac{\pi}{3}}$

27. $\frac{3}{(\cos x)(\sin x)} = \left(2\sqrt{3}\right)(\csc x)$

$\frac{3}{(\cos x)(\sin x)} = \left(2\sqrt{3}\right)(\csc x)$

$\frac{3}{(\cos x)(\sin x)} = \frac{2\sqrt{3}}{\sin x}$

$3 \sin x = 2\sqrt{3}(\cos x)(\sin x)$

$3 = 2\sqrt{3}(\cos x)$

$\cos x = \frac{\sqrt{3}}{2}$

$\boxed{x = \frac{\pi}{6}}$

28. $(\cos x)(\cot x) - (\cos x) = 0$

$(\cos x)(\cot x) - (\cos x) = 0$

$(\cos x)\frac{(\cos x)}{(\sin x)} = (\cos x)$

$\frac{(\cos^2 x)}{(\sin x)} = (\cos x)$

$\frac{\cos x}{\sin x} = 1$ $\quad \cot x = 1$ $\quad \frac{1}{\tan x} = 1$ $\quad \tan x = 1$ $\quad \boxed{x = \frac{\pi}{4}}$

Solve the following word problems. **(5 points each)**

29. A company that sells seasonal products forecasts monthly sales over a two year period to be

$$S = 25.1 + 0.44t + 4.3\sin\frac{\pi t}{6}$$

where S is measured in thousands of units and t is the time in months, with $t = 1$ representing January 2004. Estimate the sales for the following months

a) Feb 2004
b) Feb 2005
c) Sept 2004
d) Sept 2005

a) Feb 2004 $t = 2$ $S = 25.1 + 0.44(2) + 4.3\sin\left(\frac{2\pi}{6}\right) = 25.98 + (4.3)(0.8660) = 29.704$ $\boxed{29{,}704}$

b) Feb 2005 $t = 14$ $S = 25.1 + 0.44(14) + 4.3\sin\left(\frac{14\pi}{6}\right) = 31.26 + (4.3)(0.8660) = 34.984$ $\boxed{34{,}984}$

c) Sept 2004 $t = 9$ $S = 25.1 + 0.44(9) + 4.3\sin\left(\frac{9\pi}{6}\right) = 29.06 + (4.3)(-1.0000) = 24.760$ $\boxed{24{,}760}$

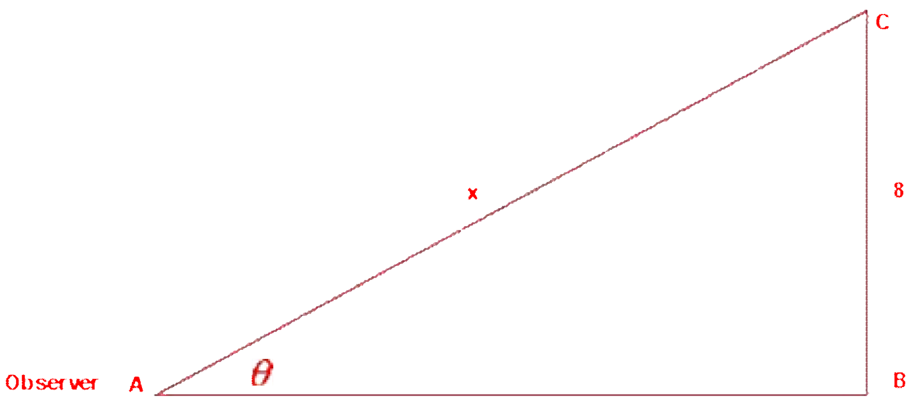
d) Sept 2005 $t = 21$ $S = 25.1 + 0.44(21) + 4.3\sin\left(\frac{21\pi}{6}\right) = 34.34 + (4.3)(-1.0000) = 30.040$ $\boxed{30{,}040}$

30. An airplane, flying at an altitude of 8 miles, is on a flight path that passes directly over an observer. If θ is the angle of elevation from the observer to the plane, find the distance from the observer to the plane when

a) $\theta = 30°$

b) $\theta = 90°$

c) $\theta = 120°$

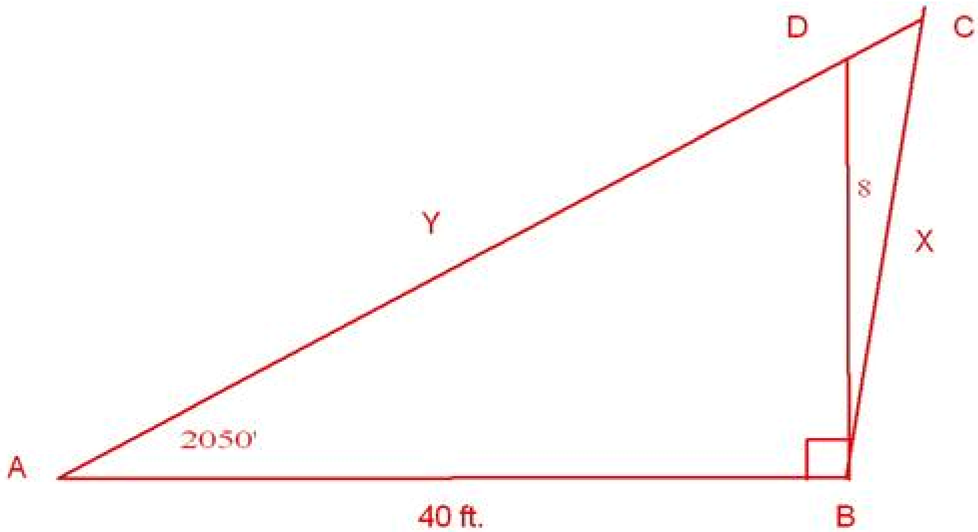


a) For $\theta = 30°$ $\sin(30) = \frac{8}{x}$ $x = \frac{8}{\sin(30)} = \frac{8}{0.500} = \boxed{16 \text{ miles}}$

b) For $\theta = 90°$ $\sin(90) = \frac{8}{x}$ $x = \frac{8}{\sin(90)} = \frac{8}{1.0} = \boxed{8 \text{ miles}}$ The plane is directly overhead.

c) For $\theta = 120°$ $\sin(120) = \frac{8}{x}$ $x = \frac{8}{\sin(120)} = \frac{8}{0.8660} \approx \boxed{9.24 \text{ miles in the opposite direction of a)}}$

31. **You are standing 40 meters from the base of a tree that is leaning 8° from vertical away from you. The angle of elevation from your feet to the top of the tree is 20°50'. Find the slant height of the tree.**



$\measuredangle ABC = 90 + 8 = 98$ and $\measuredangle ACB = 180 - 98 - 20°50' = 61°10' = 61.167$

Using the Law of Sines $\dfrac{\sin(61.167)}{40} = \dfrac{\sin(20.833)}{x}$

$\dfrac{\sin(61.167)}{40} = \dfrac{\sin(20.833)}{x}$ $\qquad x = \left(\dfrac{40}{\sin(61.167)}\right)(\sin(20.833))$

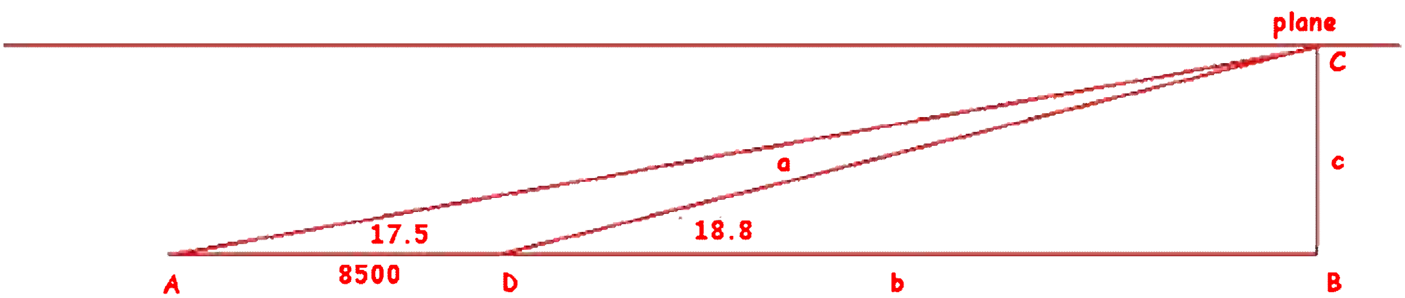
$x = \left(\dfrac{40}{0.8760}\right)(0.3556) = \boxed{16.237}$

32. A pilot has just started on the glide path for landing at an airport where the length of the runway is 8,500 feet. The angles of depression from the plane to the ends of the runway are 17.5° and 18.8°.

a) Find the air distance the plane must travel until touching down at the end of the runway.

b) Find the ground distance the plane must travel until touching down.

c) Find the altitude of the plane when the pilot begins the descent.



a) $\measuredangle ADC = 180 - 18.8 = 161.2$ and $\measuredangle ACD = 180 - 161.2 - 17.5 = 1.3$

Using the Law of Sines

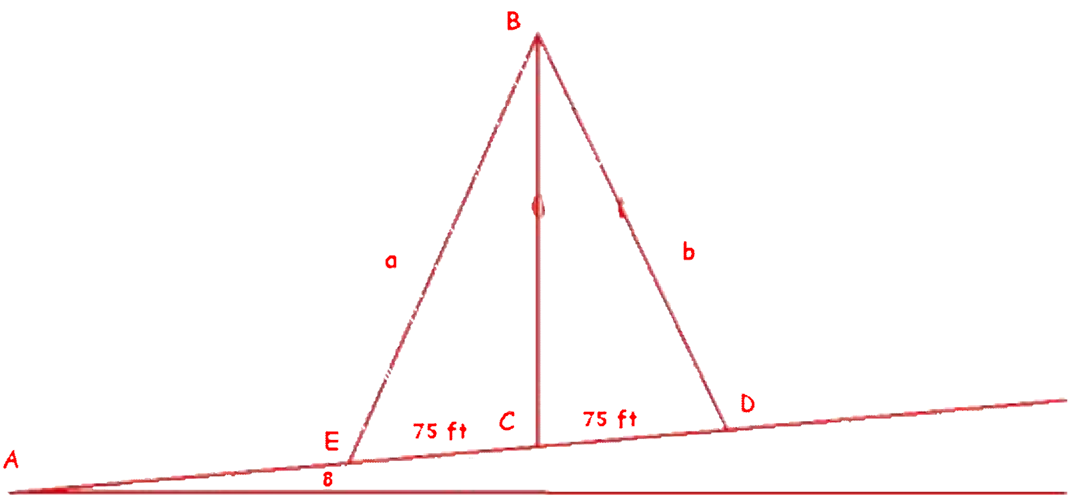
$$\frac{\sin(1.3)}{8500} = \frac{\sin(17.5)}{a} \qquad a = \frac{(8500)\sin(17.5)}{\sin(1.3)} = \frac{(8500)(0.3007)}{0.0227} = \boxed{112,596.90}$$

b) Now $\triangle DBC$ is a right triangle so ordinary trig functions can be used.

$$\cos(18.8) = \frac{b}{112,596.90} \qquad b = \cos(18.8)(112,596.90) = (0.9466)(112,596.90) = \boxed{106,584.20}$$

c) $$\sin(18.8) = \frac{c}{112,596.90} \qquad c = \sin(18.8)(112,596.90) = (0.3223)(112,596.90) = \boxed{36,289.99}$$

33. A 100 foot vertical tower is erected on the side of a hill that makes a 8° angle with the horizontal. Find the length of each of two guy wires that will be anchored 75 feet uphill and downhill from the base of the tower.



$\measuredangle BCD = 90 - 8 = 82 \qquad \measuredangle ECB = 180 - 82 = 98$

Using the Law of Cosines

$a^2 = 75^2 + 100^2 - 2 \cdot 75 \cdot 100 \cdot \cos(98)$ for $\triangle ECB$

$a^2 = 75^2 + 100^2 - 2 \cdot 75 \cdot 100 \cdot (-0.13917) = 5{,}625 + 10{,}000 + 2{,}087.597 = 17{,}712.597$

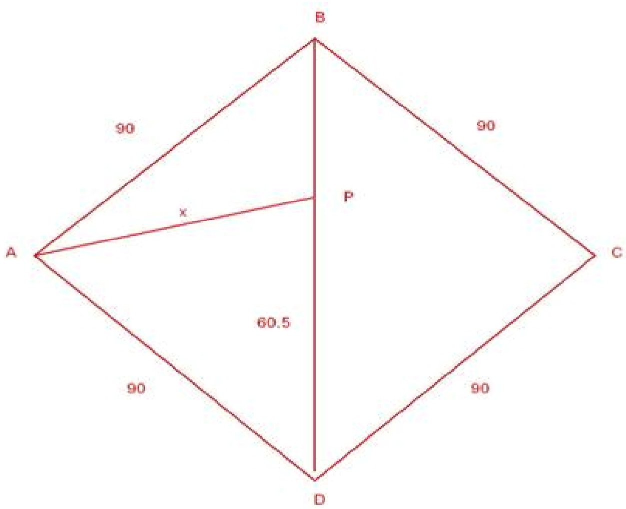
$a = \sqrt{17{,}712.597} \approx \boxed{133.09}$

$b^2 = 75^2 + 100^2 - 2 \cdot 75 \cdot 100 \cdot \cos(82)$ for $\triangle BCD$

$b^2 = 75^2 + 100^2 - 2 \cdot 75 \cdot 100 \cdot (0.139173) = 5{,}625 + 10{,}000 - 2{,}087.597 = 13{,}537.403$

$b = \sqrt{13{,}537.403} \approx \boxed{116.35}$

34. On a baseball diamond with 90 foot sides, the pitcher's mound is 60.5 feet from home plate. How far is it from the pitcher's mound to third base?



It is important to note that $\overline{AP}$ IS NOT $\perp$ $\overline{BD}$ but since $\square ABCD$ is a square so $\overline{BD}$ bisects $\angle ABC$ and $\angle CDA$ so each of the four smaller angles are 45°

So using the Law of Cosines

$$x^2 = 90^2 + (60.5)^2 - 2 \cdot 90 \cdot 60.5 \cdot \cos(45)$$
$$= 8{,}100 + 3{,}660.25 - (10{,}890)(0.7071)$$
$$= 8{,}100 + 3{,}660.25 - 7{,}700.32 = 4{,}059.93$$
$$x = \sqrt{4{,}059.93} \approx \boxed{63.71 \text{ feet}}$$