Math 275
May 14, 2008

Final Examination

Name ______________________________

This test consists of 100 points on 10 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work. Check the box to indicate you read these instructions. ☐

(10) 1. Find an equation of the plane passing through the points $(-1, 2, 0)$, $(2, 0, 1)$ and $(-5, 3, 1)$.

(10) 2. Find the line of intersection between the planes $x + y - z = 1$ and $2x - 3y + 4z = 5$. You may use a vector equation, parametric equations or symmetric equations for the line.

(10) 3. The arclength of a curve C can be computed using the line integral $\int_C 1\, ds$. Find the length of the curve traced out by $\mathbf{r}(t) = 2t^{(3/2)}\mathbf{i} + \cos(2t)\mathbf{j} + \sin(2t)\mathbf{k}$ as t goes from 0 to 1.

(10) 4. The helix $\mathbf{r}_1(t) = \cos(t)\mathbf{i} + \sin t\mathbf{j} + t\mathbf{k}$ intersects the curve $\mathbf{r}_2(t) = (1+t)\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ at the point $(1, 0, 0)$. Find the angle of intersection between these curves at this point.

5. A student was asked to find the equation of the tangent plane to the surface $z = x^3 - y^2$ at the point where $(x, y) = (2, 3)$. His answer was

$$z = 3x^2(x - 2) - 2y(y - 3) - 1$$

(3) (a) At a glance, how do you know this is wrong?

(3) (b) What mistake did this student make?

(4) (c) Answer the question correctly.

(10) 6. Find the maximum and minimum values of the scalar field

$$f(x, y, z) = x - 2y + 2z$$

on the surface of the sphere

$$x^2 + y^2 + z^2 = 1$$

(Hint: This is a Lagrange multipliers problem.)

7. The triple integral

$$\iiint\limits_S f(x, y, x)\, dV$$

of a positive function reduces to the iterated integral

$$\int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x, y, z)\, dz\, dy\, dx$$

(5) (a) Describe the region S by means of a sketch

(5) (b) Express the triple integral as one of more iterated integrals in which the first integration is with respect to y

(10) 8. Evaluate the following iterated integral

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} \frac{1}{1+x^2+y^2}\, dx\, dy$$

(Hint: Sketch the region of integration and do a change of variables.)

(10) 9. Let $\mathbf{F}(x, y) = (2x + y^2 + 3x^2y)\mathbf{i} + (2xy + x^3 + 3y^2)\mathbf{j}$. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is the arc of the curve $y = x\sin(x)$ from $(0, 0)$ to $(\pi, 0)$ (You might want to show $\mathbf{F}$ is a gradient and then use the fundamental theorem of line integrals.)

(10) 10. Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = xz\mathbf{i} - 2y\mathbf{j} + 3x\mathbf{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 4$ with outward orientation.