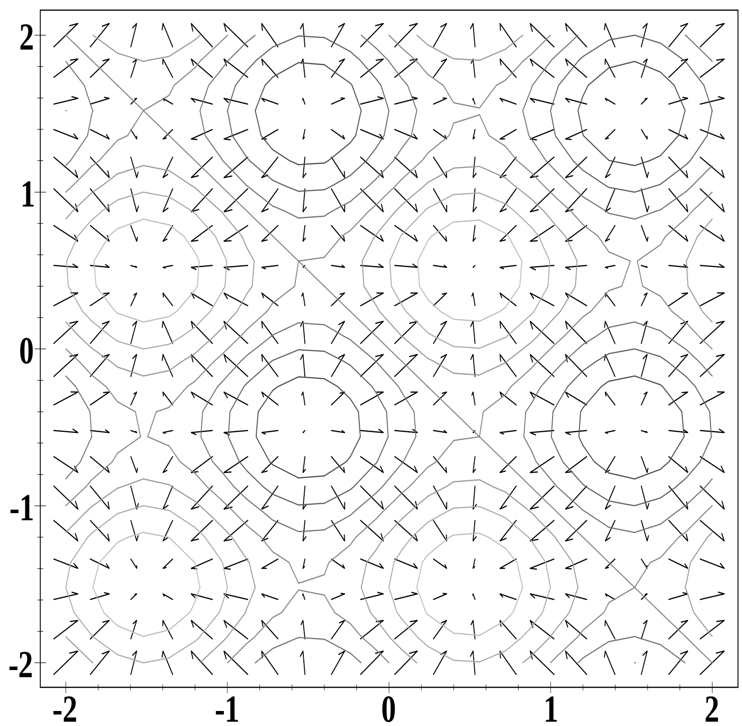
M234–001
May 14, 1998

Final Examination Name ______________________

This test consists of **9** pages, all different and none intentionally left blank.
Take a minute *right now* to ensure that you have all 9 of these pages.
In order to receive credit for your answers, you must show your work!!

1. (10 points) The following graph shows the level curves and the vector field for the gradient of a function $z = f(x, y)$ over the square $-2 \le x \le 2$ and $-2 \le y \le 2$. Is the point $(1/2, 1/2, f(1/2, 1/2))$ a maximum or a minimum? How about the point $(-1/2, 3/2, f(-1/2, 3/2))$?



2. (3 points each) Match the following vector fields with their graphs. No reasons need to be given.

   a. $y\mathbf{i} + x\mathbf{j}$

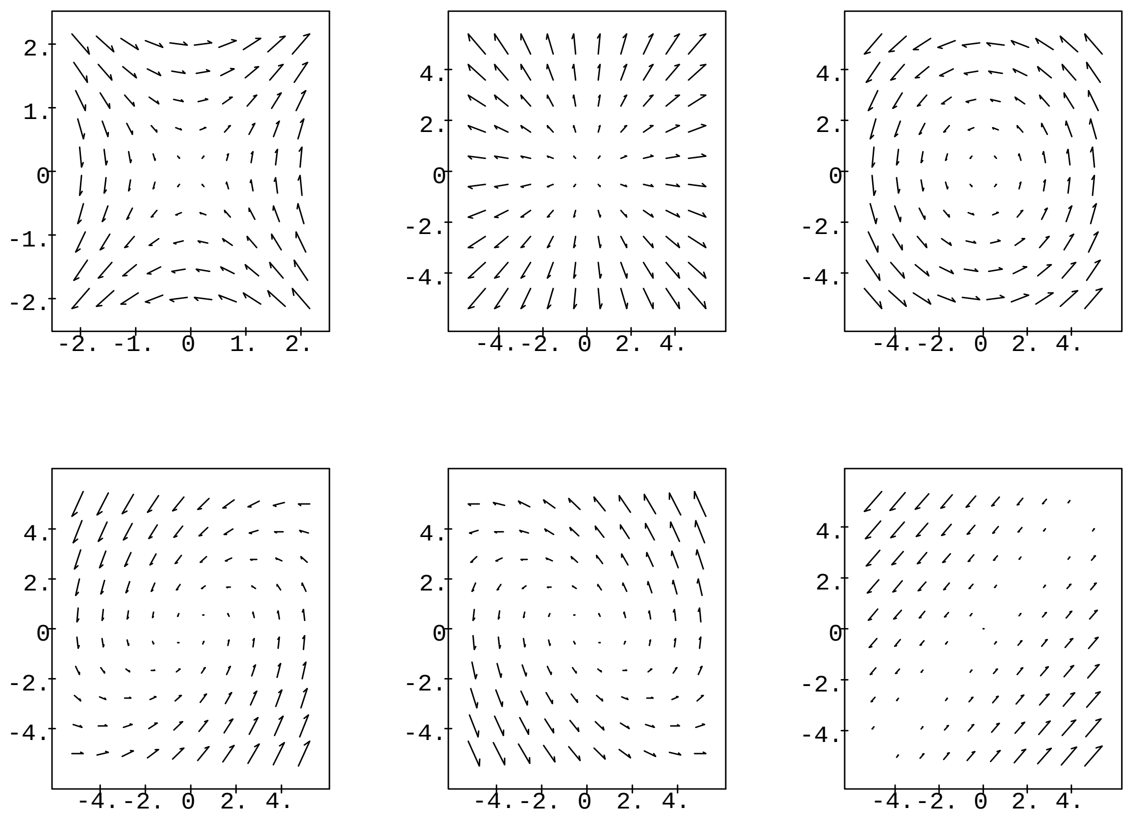
   b. $-y\mathbf{i} + x\mathbf{j}$

   c. $-y\mathbf{i} + (x + y)\mathbf{j}$

   d. $x\mathbf{i} + y\mathbf{j}$

   e. $-y\mathbf{i} + (x - y)\mathbf{j}$

   f. $(x - y)\mathbf{i} + (x - y)\mathbf{j}$



3. (10 points) Let C be the curve $\mathbf{r}(t) = (2t^3/3)\mathbf{i} + (1 - 2 * t^2)\mathbf{j} + 4t\mathbf{k}$ for $0 \leq t \leq 3$. Find the length of C.

4. (5 points each) Let C be defined by the $\mathbf{r}(t) = \langle -1 + 5\sin(t), 3\cos(t), 1 - 4\cos(t) \rangle$ for $t \in [0, 2\pi]$. Find the unit tangent, normal and binormal vectors for C.

5. (10 points) Prove that

$$\lim_{(x,y)\to(0,0)} \frac{x^3 - 2y^3}{3x^3 + 4y^3}$$

does not exist.

6. The original problem was too hard. Lets try this one instead.

   Let $\mathbf{u} = \langle 2, -1, 5 \rangle$ and let $\mathbf{v} = \langle 1, -2, -3 \rangle$

   a. (5 points) Find $\mathbf{u} \cdot \mathbf{v}$

   b. (5 points) Find $\mathbf{u} \times \mathbf{v}$

   c. (5 points) Find the component of $\mathbf{u}$ in the direction of $\mathbf{v}$

   d. (5 points) Find the parametric equations of a line perpendicular to both $\mathbf{u} + \mathbf{v}$ and $\mathbf{u} - \mathbf{v}$ which passes through the point $(1, 1, 1)$

7. (10 points) If $z = \ln\left(\sqrt{\frac{x+y}{x-y}}\right)$, $x = t + \frac{1}{t}$ and $y = t - \frac{1}{t}$, find $\frac{dz}{dt}$

8. (10 points) Find $f(x, y)$ if $f(1, 1) = 1$ and

$$\nabla f(x, y) = \left\langle \frac{1 - 2x^2}{x}, \frac{2y^2 - 1}{y} \right\rangle$$

9. (10 points) Find an equation of the tangent plane and the normal line to the graph of $z = x^2 - xy - 2y^2$ at the point $(2, -1, 4)$.

10. (10 points) Evaluate $\iint\limits_R (x+y)/y^2 \, dA$ where R is the region bounded by $x = y^2$, $x = y+2$, $y = 1$ and $y = 2$.

11. (20 points) Evaluate $\displaystyle\iint\limits_R (x - 4y) \sin\left(\frac{\pi(x-y)}{2}\right) dA$ where R is the region enclosed by the parallelogram with vertices $(-2, -2)$, $(2, -1)$, $(2, 2)$, and $(6, 3)$. Use a suitable change of variables.

12. (5 points for picture, 15 for integration) A solid occupies the first-octant region bounded by the surfaces $y = x^2$, $y = z$, $y = 1$, $y = 4$ and $z = 0$. If its density is given by $\rho(x, y, z) = (x + z)/\sqrt{y}$. Find its center of mass.

13. (10 points) Let $\mathbf{F}(x, y, z) = \langle 2xyz, x - y, y + 2z \rangle$ Find the divergence and curl of $\mathbf{F}$

14. (10 points) Let C_1 be the quarter of the unit circle from $(1, 0)$ to $(0, 1)$ and let C_2 be the line segment from $(0, 1)$ to $(0, -1)$. Let $C = C_1 \cup C_2$. Let $f(x, y) = (x + y)^2$ and find the line integral $\int_C f(x, y)ds$.

15. (12 points) Use Green's theorem to evaluate

$$\iint_R (3x^2 - 2y)dA$$

where R is the triangular region with vertices (0,0), (1,0) and (1,1).

16. (15 points) Compute the value of the surface integral $\iint_S \mathbf{F} \cdot \mathbf{n}\ dS$ where $\mathbf{F}(x, y, z) = x\mathbf{i} - xz\mathbf{j} + 3z\mathbf{k}$ and S is the surface defined by $y = \sqrt{x^2 + z^2}$ for $x^2 + z^2 \leq 4$.