Math 275-030
July 30, 2009

Exam Four

This test consists of 100 points and 6 questions. Take a few seconds right now to be sure you have all the questions. The point value of each question is to the left of the question number. Show all your work in the bluebook as specified. You should not start a problem on the back of a page. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

(16) 1. (Page 1) A wire in the shape of the quarter circle $\mathbf{r}(u) = 3\cos(u)\mathbf{i} + 3\sin(u)\mathbf{j}$, $0 \le u \le \frac{\pi}{2}$ has varying mass density $\lambda(x, y) = 3x + 2y$. Find the total mass of the wire.

(16) 2. (Page 2) Show the the vector field $\mathbf{F} = y^2z^3\mathbf{i} + 2xyz^3\mathbf{j} + 3xy^2z^2\mathbf{k}$ is a gradient and find the potential for $\mathbf{F}$.

(16) 3. (Page 3) Use Green's theorem to evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F} = (1 + 10xy + y^2)\mathbf{i} + (6xy + 5x^2)\mathbf{j}$ and C is the square with vertices $(0, 0)$, $(a, 0)$, (a, a), $(0, a)$.

(16) 4. (Page 4) Determine the flux of $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ out of the cylindrical surface S described by

$$\mathbf{r}(u, v) = 2\cos(u)\mathbf{i} + 2\sin(u)\mathbf{j} + v\mathbf{k}$$
$$0 \le u \le 2\pi, \; 0 \le v \le 1$$

(20) 5. (Page 5) Let $\mathbf{F} = \frac{1}{2}y\mathbf{i} + 2xz\mathbf{j} - 3x\mathbf{k}$ and let S be the surface $y = 1 - (x^2 + z^2)$ from $y = -8$ to $y = 1$. Calculate the flux of $\nabla \times \mathbf{F}$ in the direction of the unit normal vector $\mathbf{n}$ which has positive $\mathbf{j}$ component.

(16) 6. (Page 6) Use the divergence theorem to find the flux of

$$\mathbf{F} = \ln(x^2 + y^2)\mathbf{i} + \left(\frac{2z}{x}\tan^{-1}\left(\frac{y}{x}\right)\right)\mathbf{j} + z\sqrt{x^2 + y^2}\mathbf{k}$$

where D is the region between the cylinders $x^2 + y^2 = 1$ and $x^2 + y^2 = 2$ for $1 \le z \le 2$.