Math 275-030 **Exam 4** Name ____________________
July 26, 2007

This test consists of 100 points and 8 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

(7) 1. Let $T(u,v) = \frac{u-v}{\sqrt{2}}\mathbf{i} + \frac{u+v}{\sqrt{2}}\mathbf{j}$. What is the image of $S = \{(u,v) \mid 0 \leq x \leq 1, \quad 0 \leq y \leq 1\}$ under the transformation T?

(7) 2. Use a change of variables to Evaluate

$$\iint_D (2x+y)^2 e^{x-y}\, dA$$

where D is the region enclosed by the lines $2x+y=1$, $2x+y=4$, $x-y=-1$, and $x-y=1$

(15) 3. Find $\int_C x^2y\,dx - xy\,dy$ where C is the curve with equation $y^2 = x^3$ from $(-1, 1)$ to $(1, 1)$.

(14) 4. Evaluate

$$\oint_C (x^2 - y^2)\, dx + (x^2 + y^2)\, dy$$

where C is the boundary of the square with vertices (0, 0), (1, 0), (1, 1), and (0, 1) oriented *clockwise*.

5. Let $\mathbf{X} : \mathbb{R}^2 \to \mathbb{R}^3$ be the parameterized surface given by

$$\mathbf{X}(s,t) = \langle s^2 - t^2, s + t, s^2 + 3t \rangle$$

(7) (a) Determine a normal vector to this surface at the point

$$(3, 1, 1) = \mathbf{X}(2, -1)$$

(7) (b) Find an equation of the plane tangent to this surface at the point $(3, 1, 1)$

6. Let S be the funnel-shaped surface defined by $x^2 + y^2 = z^2$ for $1 \le z \le 9$ and $x^2 + y^2 = 1$ for $0 \le z \le 1$

(5) (a) Sketch the surface S.

(5) (b) Determine the outwardly pointing normals to S

(5) (c) Evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = -y\mathbf{i} + x\mathbf{j} + z\mathbf{k}$ and S is oriented by outwardly pointing normals.

(14) 7. Let S be the silo shaped surface defined by the union of the two surfaces S_1 and S_2 where S_1 is defined by

$$x^2 + y^2 = 9, \quad 0 \le z \le 8$$

and S_2 is defined by

$$x^2 + y^2 + (z-8)^2 = 9, \quad z \ge 8$$

Using the outwardly directed normal, find

$$\iint_S (\operatorname{curl}(\mathbf{F}) \cdot d\mathbf{S}$$

where $\mathbf{F} = (x^3 + xz + yz^2)\mathbf{i} + (xyz^3 + y^7)\mathbf{j} + x^2z^5\mathbf{k}$

(14) 8. Use the divergence theorem to find the flux of $\mathbf{F}(x, y, z) = (2x - 3y)\mathbf{i} + (4y + 2z)\mathbf{j} + (x + z)\mathbf{k}$ through the surface of the sphere $x^2 + y^2 + z^2$ with outwardly directed normal.