Math 275-030 — **Exam 3**

July 16, 2009

> This test consists of 100 points and 7 questions. Take a few seconds right now to be sure you have all the questions. The point value of each question is to the left of the question number. Show all your work in the bluebook as specified. As a general rule, you should not start a problem on the back of a page. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

(14) 1. (Page 1) Evaluate the following integral

$$\int_{-1}^{1}\int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \frac{2}{(1+x^2+y^2)^2}\,dy\,dx$$

(15) 2. (Page 2) Find the volume of the solid which is below the paraboloid $z = x^2 + y^2$ and above the region in the xy-plane which is enclosed by the graphs of $y = 6 - x^2$ and $y = x$

(14) 3. (Page 3) Evaluate the following integral

$$\int_{0}^{4}\int_{y/2}^{2} e^{x^2}\,dx\,dy$$

(14) 4. (Page 4) Find the volume of the region bounded by $z = x^2 + y^2$ and $z = 10 - x^2 - 2y^2$.

(14) 5. (Page 5) Let $W = \{(x, y, z)|\sqrt{x^2+y^2} \le z \le 1\}$. Find the appropriate limits a, b, $\phi_1(x)$, $\phi_2(x)$, $\gamma_1(x, y)$, and $\gamma_2(x, y)$ and write the triple integral over the region W as an iterated integral of the form

$$\iiint_W f(x,y,z)\,dV = \int_a^b \left\{ \int_{\phi_1(x)}^{\phi_2(x)} \left[\int_{\gamma_1(x,y)}^{\gamma_2(x,y)} f(x,y,z)\,dz \right] dy \right\} dx$$

(14) 6. (Page 6) Let S be the region between the spheres $x^2 + y^2 + z^2 = 4$ and $x^2 + y^2 + z^2 = 9$. Evaluate the following triple integral

$$\iiint_S \frac{dV}{(x^2+y^2+z^2)^{3/2}}$$

7. (Page 7) Let R be the region bounded by the graphs of $x + y = 1$, $x + y = 4$, $x - y = -1$ and $x - y = 1$. Let $u = x + y$ and $v = x - y$.

(5) (a) Find x and y in terms of u and v.

(5) (b) Find the Jacobian of the transformation found in part a) from the uv-plane to the xy plane.

(5) (c) Use this change of variables to compute

$$\iint_R (x+y)^2 e^{x-y}\,dA$$