

This test consists of 100 points and 4 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

NO GRAPHING CALCULATORS

- (10) 1. Let $f(x, y) = x^2y^2 + 2xy$, and let $\mathcal{R} = \{(x, y) : 0 \leq x \leq 2, -1 \leq y \leq 1\}$. Partition the rectangle into 4 equal subrectangles. Using the midpoint of the rectangles, compute the Riemann sum which approximates

$$\iint_{\mathcal{R}} f(x, y) \, dA$$

- (20) 2. A plate has the shape of the region bounded by $y = x^2$ and $x = y^2$ in the first quadrant. The density of this plate is given by $\delta(x, y) = xy^2$. Find the moment about the x axis for this plate.

3. When the double integral

$$\iint_E f(x, y) \, dA$$

is set up as an iterated integral, you get

$$\int_1^3 \int_{-x^2}^{x^2} f(x, y) \, dy \, dx$$

(10) (a) Sketch the region E .

(10) (b) Rewrite the iterated integral so you integrate with respect to x first.

- (20) 4. Convert the following integral to polar coordinates and evaluate:

$$\int_1^3 \int_{-x}^x \frac{1}{\sqrt{x^2 + y^2}} dy dx$$

- (15) 5. Sketch the region of integration for the following iterated integral

$$\int_{-2}^2 \int_0^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{8-x^2-y^2}} f(x, y, z) dz dy dx$$

- (15) 6. When computing the moments for a region E in \mathbb{R}^3 (3-space), you computed the following values: Moment about the xy -plane is 15, Moment about the xz -plane is 21, Moment about the yz -plane is 18. The total mass of the region is 3. What are the coordinates $(\bar{x}, \bar{y}, \bar{z})$ of the center of mass of the region E .