Math 275
April 14, 2003

Exam 3 Name _____

This test consists of 8 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

(10) 1. The following table gives values of $f(x,y) = e^{-(x^2+y^2)}$ at various points in the rectangle $R = [0,1] \times [0,1]$

		x				
		0.0	0.25	0.50	0.75	1.00
	0.0	1	0.9394	0.7788	0.5698	0.3679
	0.25	0.9394	0.8825	0.7316	0.5353	0.3456
y	0.50	.7788	.7316	0.6065	0.4437	0.2865
	0.75	0.5698	0.5353	0.4437	0.3247	0.2096
	1.00	0.3679	0.3456	0.2865	0.2096	0.1353

Using the partition $0 = x_0 < x_1 = 0.5 < x_2 = 1$ and $0 = y_0 < y_1 = 0.5 < y_2 = 1$, find an approximation to

$$\iint_R e^{-(x^2+y^2)}\, dA$$

(10) 2. A solid is formed above the rectangle $R = [0, 2] \times [0, 4]$ by the graph of $f(x, y) = 2 + xy$. Using Reimann sums with 4 subdivisions (a total of 4 sub rectangles, not four divisions on a side.) Find upper and lower bounds for the volume of this solid.

(10) 3. For the following problem, sketch the region of integration, R, and evaluate the iterated integral

$$\iint_R \sin(x)\, dA = \int_1^5 \int_x^{2x} \sin(x)\, dy\, dx$$

(10) 4. Sketch the region of integration and evaluate the integral

$$\int_0^1 \int_{\sqrt{y}}^1 \sqrt{1+x^3}\, dx\, dy$$

(You may wish to change the order of integration.)

(10) 5. When solving $\iint_R f(x,y)dA$ the following sum of iterated integrals was obtained.

$$\iint_R f(x,y)dA = \int_{-4}^0 \int_0^{2x+8} f(x,y)\, dy\, dx + \int_0^4 \int_0^{-2x+8} f(x,y)\, dy\, dx$$

Sketch the region R and set up the iterated integral where you integrate with respect to x first.

(10) 6. Set up, but do not evaluate, an iterated integral for the volume of the solid which lies below the graph of $f(x, y) = 25 - x^2 - y^2$ and above the plane $z = 16$

(10) 7. Convert the following integral to polar coordinates and evaluate:

$$\int_0^{\sqrt{2}} \int_y^{\sqrt{4-y^2}} xy \, dx \, dy$$

8. Let Ω be the region bounded by the cardioid (polar coordinates) $r = 1 + \cos\theta$, and suppose the density $\rho(x, y)$ is proportional to the distance between (x, y) and the origin $(0, 0)$.

(5) (a) What is the moment about the x axis of the region Ω?

(Question 8 continued.)

(5) (b) What is the moment about the y axis of the region Ω?

(Sill Question 8.)

(5) (c) What is the total mass of Ω?

(5) (d) Using your answers to the previous parts, what are the coordinates of the center of mass of the region Ω?

(10) 9. What is the average value of $f(x,y) = e^{x+y}$ over the region

$$R = \{(x,y) \mid 0 \le x \le 1,\ x-1 \le y \le x+1\}$$