Math 275 **Exam 3** Name ____________________

April 11, 2002

> This test consists of 5 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

1. When a double integral was set up for the volume V under the paraboloid $z = x^2 + y^2$ and above the region S in the xy-plane, the following sum of iterated integrals was obtained.

$$V = \int_0^1 \int_0^y (x^2 + y^2)\, dxdy + \int_1^2 \int_0^{2-y} (x^2 + y^2)\, dxdy$$

(10) (a) Sketch the region S in the xy-plane.

(10) (b) Express V as an iterated integral (or the sum of iterated integrals) in which the order of integration is reversed.

(10) (c) Compute V.

(10) 2. A thin plate is bounded by an arch of the parabola $y = 2x - x^2$ and the interval $0 \leq x \leq 2$. Determine its mass if the density at each point is given by $\rho(x, y) = (1 - y)/(1 + x)$

(10) 3. Make a sketch of the region $S = \{(x, y) \mid x^2 + y^2 \leq 2x\}$ and express the double integral

$$\iint_S f(x, y)\, dx\, dy$$

as an iterated integral in polar coordinates.

4. The triple integral $\iiint\limits_E f(x,y,z)\,dV$ of a positive function f reduces to the iterated integral

$$\int_0^1 \int_0^{1-x} \int_0^{x+y} f(x,y,z)\,dz\,dy\,dx$$

(10) (a) Describe the region E by means of a sketch, showing its projection on the xy-plane.

(10) (b) Express the triple integral as one or more iterated integrals in which the first integration is with respect to y.

5. Let E be the region bounded above by the sphere $x^2 + y^2 + z^2 = 4$ and below by the cone $z = \frac{\sqrt{3}}{3}\sqrt{x^2 + y^2}$, and let $f(x, y, z) = x + y + z$.

(10) (a) Sketch the region E.

(10) (b) Evaluate $\iiint\limits_E f(x, y, z)\, dV$

(10) 6. Find the area of the surface cut from the paraboloid $x^2 + y^2 = z$ by the plane $z = 2$.