M275–001
November 2, 1999 TEST #3 Name _____

This test consists of **7** pages, all different and none intentionally left blank.
Take a minute *right now* to ensure that you have all 7 of these pages.
In order to receive credit for your answers, you must show your work!!

1. (15 points) Let $f(x,y) = x + y$ on the rectangle $R = \{(x,y) | 1 \leq x \leq 2, \ \ 2 \leq y \leq 3\}$ Using the partitions $x_0 = 1$, $x_1 = \frac{3}{2}$, $x_2 = 2$ and $y_0 = 2$, $y_1 = \frac{5}{2}$, $y_2 = 3$, find the Riemann sum approximating

$$\iint_R f(x,y)\, dA$$

using the upper left corners of the rectangles created by these partitions.

2. Consider the iterated integral

$$\int_0^1 \int_{y^2}^{2y} f(x,y)\ dx\ dy$$

(a) (5 points) Sketch the region of integration for the integral.

(b) (15 points) Write the corresponding iterated integral(s) over the region in part a, integrating with respect to y first.

3. (15 points) A lamina has the shape of the outer loop of the graph of the polar coordinate curve $r = \frac{1}{2} - \cos(\theta)$ and density given by $\rho(x, y) = x + \frac{3}{2}$ Set up an integral (including accurate limits of integration) whose value is the moment about the x-axis for the lamina. (Remember that your solution MUST contain a sketch of the lamina.)

4. (15 points) Set up an integral (including accurate limits of integration) whose value is the volume of the region bounded above by the (upside-down) cone $z = 4 - \sqrt{x^2 + y^2}$ and below by the plane $z = -5$. Once again, your solution must contain a sketch of the solid showing how you found your limits of integration.

5. Consider the following integral

$$\iint_R (x - 2y)\sin(2x + y)\, dA$$

where R is the parallogram in the xy-plane with vertices $(1, 1)$, $(3, 2)$ $(2, -1)$, and $(4, 0)$.

a. (5 points) Sketch the region R in the xy-plane.

b. (5 points) If $u = x - 2y$ and $v = 2x + y$ is the change of coordinates transformation, what is the Jacobian of the transformation?

(Problem 5 continued.)

c. (5 points) Sketch the region in the uv-plane corresponding to the region R under this transformation?

d. (5 points) Find the value of the integral by a change to the uv coordinate system and evaluating the transformed integral.

6. (15 points) Convert the following integral to spherical coordinates. (DO NOT EVALUATE the integral.) Remember to include a sketch of the region as well as your computations to determine the limits in spherical coordinates.

$$\int_{-\frac{3}{2}}^{\frac{3}{2}} \int_{-\sqrt{\frac{9}{4}-x^2}}^{\sqrt{\frac{9}{4}-x^2}} \int_{\frac{1}{\sqrt{3}}\sqrt{x^2+y^2}}^{\sqrt{3-x^2-y^2}} xyz \; dzdydx$$