

This test consists of **5** pages, all different and none intentionally left blank.

Take a minute *right now* to ensure that you have all 5 of these pages. In order to receive credit for your answers, you must show your work!!

1. Give an equivalent integral with the order of integration reversed.

$$\int_0^4 \int_{y^2/4}^{2\sqrt{y}} f(x, y) dx dy$$

2. Change the following integral to polar coordinates. This requires that you draw the region of integration. Do not integrate!

$$\int_0^1 \int_x^{\sqrt{2-x^2}} (x^2 + y^2)^2 dy dx$$

3. Set up, but do not evaluate, the integrals necessary to find the center of mass of the lamina that occupies the region $R = \{(x, y) | 0 \leq x \leq 1, x \leq y \leq 1\}$ with density $\rho(x, y) = 2y$

4. Evaluate the following integral by making an appropriate change of variables

$$\int_0^{1/2} \int_y^{1-y} \frac{\sin(x-y)}{\cos(x+y)} dx dy$$

5. Change the following integral to an equivalent one in spherical coordinates or cylindrical coordinates. (Do not evaluate).

$$\int_0^1 \int_{-\sqrt{1-z^2}}^{\sqrt{1-z^2}} \int_{-\sqrt{1-y^2-z^2}}^{\sqrt{1-y^2-z^2}} x^2 dx dy dz$$

6. Set up an integral whose value is the (surface) area of the part of the portion of the paraboloid $z = 2 - x^2 - y^2$ that lies above the xy - plane.

7. Find and identify (as maxima, minima or saddle points, all critical points for

$$f(x, y) = x^2 - 6xy + 2y^3 - 8x - 16$$

8. Find the maximum and minimum values of $f(x, y) = x^3 + 3y^2$ subject to the constraint $xy + 4 = 0$