

This test consists of 100 points and 11 questions. Take a few seconds right now to be sure you have all the questions. The point value of each question is to the left of the question number. Show all your work in the bluebook as specified. Start problems on the front of the page. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

- (5) 1. (Top quarter of page 1) Sketch some level curves for the function $f(x, y) = \sqrt{x^2 + y^2}$. Be sure you label the values of f on these curves.

- (5) 2. (Second quarter of page 1) Find the domain of the function $f(x, y) = \ln(xy)$

- (10) 3. (Bottom half of page 1) Show that $\lim_{(x,y) \rightarrow (0,0)} \frac{x}{\sqrt{x^2 + y^2}}$ does not exist.

- (10) 4. (Top half of page 2) Find and simplify the second partial derivatives of

$$f(x, y) = x \ln(xy)$$

- (10) 5. (Bottom half of page 2) At the point $(1, 2)$ the function $f(x, y)$ has a derivative of 2 in the direction of $(2, 2)$ and a derivative of -2 in the direction of $(1, 1)$. Find the derivative of f at the point $(1, 2)$ in the direction of $(4, 6)$.

- (10) 6. (Top third of page 3) Show that if $w = f(s)$ is a differentiable function of s and if $s = y + 5x$, then

$$\frac{\partial w}{\partial x} - 5 \frac{\partial w}{\partial y} = 0$$

- (10) 7. (Middle third of page 3) What is the largest value the directional derivative of $f(x, y, z) = xyz$ can have at the point $(1, 1, 1)$

- (10) 8. (Bottom third of page 3) Find an equation of the plane which is tangent to the graph of

$$f(x, y) = \ln(x^2 + y^2)$$

at the point $(2, 1, \ln(5))$

- (10) 9. (page 4) Find and identify all local maxima, minima and saddle points for

$$f(x, y) = 5x^2 + 4xy - 2y^2 + 4x - 4y$$

- (10) 10. (Page 5) Find the maximum and minimum value of

$$f(x, y) = 4xy - x^4 - y^4 + 16$$

on the triangular region bounded by the lines $y = -2$, $y = x$ and $x = 2$

- (10) 11. (page 6) Find the extreme values of $f(x, y, z) = x(y + z)$ subject to $x^2 + y^2 = 1$ and $xz = 1$.