

This test consists of 90 points and 4 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

- (10) 1. A function  $f$  has continuous first partial derivatives. Let  $g(u, v) = f(u^2 + \cos(\pi v), v + 2 \sin(\pi u))$ . Use the following table to compute  $g_u(1, 2)$  and  $g_v(1, 2)$

	$f$	$g$	$f_x$	$f_y$
(2,2)	3	-1	3	6
(1,2)	2	-1	4	3

- (10) 2. Find the points on the surface  $xy + yz + xz - z - z^2 = 0$  where the tangent plane is parallel to the  $xy$ -plane.

3. Let  $f(x, y) = x^2 - y^2 - 2x + 4y + 1$ .
- (15) (a) Complete the square for  $f$ , identify the quadric surface and sketch it.
- (10) (b) Find the maximum and minimum values of  $f$  (above) subject to the condition  $x^2 - 2x + y^2 - 4y = 4$

4. Consider the curve

$$\mathbf{r}(t) = \left(\frac{t^3}{4} - 2\right)\mathbf{i} + \left(\frac{4}{t} - 3\right)\mathbf{j} + \cos(t - 2)\mathbf{k}$$

(5) (a) For what value of  $t$  does the curve pass through the point  $(0, -1, 1)$ ?

(5) (b) Find a vector which is tangent to the curve at that point?

(5) (c) What is a normal vector to the surface

$$x^3 + y^3 + z^3 - xyz = 0$$

at the point  $(0, -1, 1)$ .

(5) (d) Show that the tangent vector in part b lies in the tangent plane to the surface at the point  $(0, -1, 1)$

(10) 5. Is there a direction for a unit vector  $\mathbf{u}$  in which the directional derivative of  $f(x, y) = x^2 - 2xy + 4y^2$  at the point  $(1, 2)$  equals 14? Give reasons for your answer.

(8) 6. The critical values for the function  $f(x, y) = x^3 + y^3 + 3x^2 - 3y^2 - 8$  are  $(0, 0)$ ,  $(-2, 0)$ ,  $(0, 2)$ , and  $(-2, 2)$ . Identify these as local maxima, local minima or saddle points.

(7) 7. The tangent plane to the graph of a differentiable function  $f$  at the point  $(1, -1, 4)$  has equation

$$3(x - 1) + 5(y + 1) + (z - 4) = 0$$

Use this information to approximate a value for  $f(1.05, -0.95)$ .