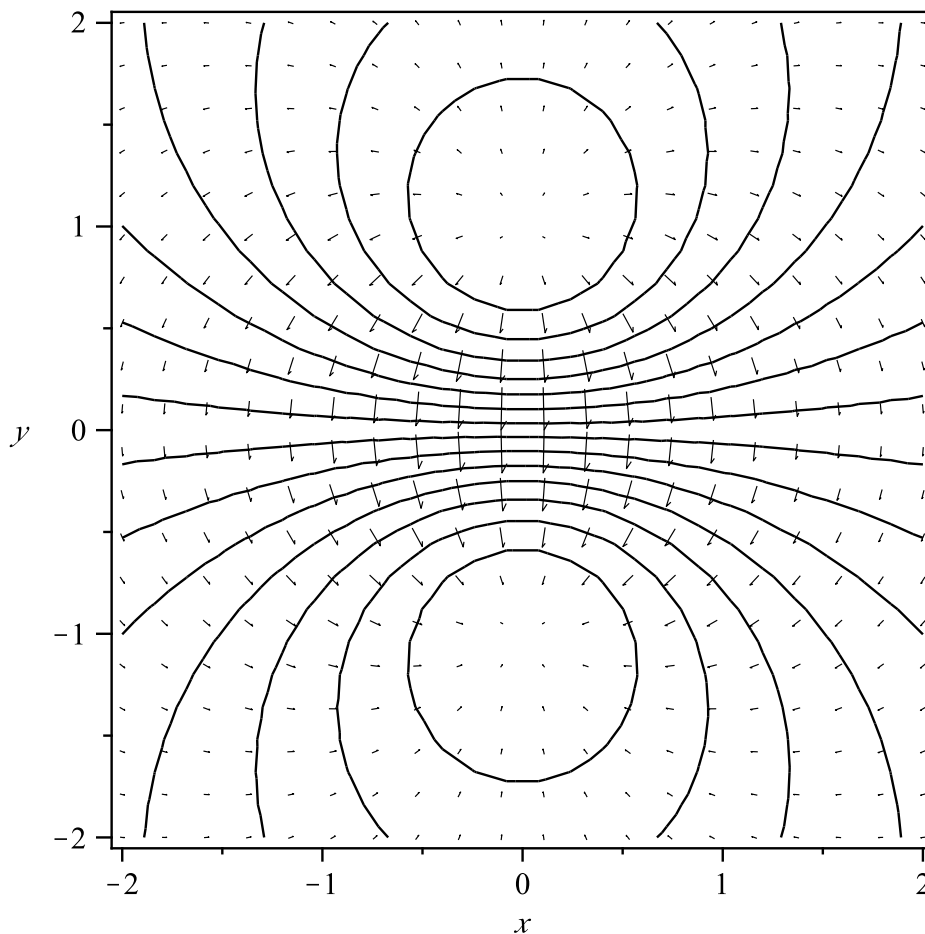


This test consists of 100 points and 5 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

### NO GRAPHING CALCULATORS

- (10) 1. The following shows is a contour graph of a function  $f$  together with the gradient of the function. Use this information to locate a local maximum and a local minimum for the function. Explain your reasoning.



2. This problem is concerned with

$$\lim_{(x,y) \rightarrow (0,0)} f(x,y)$$

where

$$f(x,y) = \frac{x^2 y}{x^4 + y^2}$$

(8) (a) Let  $y = mx$  and show that

$$\lim_{x \rightarrow 0} f(x, mx) = 0$$

so along any line through the origin,  $f$  has a limit zero.

(8) (b) Let  $y = x^2$  and compute

$$\lim_{x \rightarrow 0} f(x, x^2)$$

(8) (c) What can you conclude about

$$\lim_{(x,y) \rightarrow (0,0)} f(x,y)$$

- (10) 3. If the directional derivative of  $f$  at  $(2, 2)$  in the direction of  $\mathbf{i} + \mathbf{j}$  is 3, and in the direction of  $\mathbf{i} - \mathbf{j}$  is 2, what are the values of  $f_x(2, 2)$  and  $f_y(2, 2)$ ?

- (20) 4. Find the critical values for

$$f(x, y) = xy(1 - x - y)$$

- (18) 5. The function  $f(x, y) = 4x + 6y - x^2 - y^2$  has a critical value at  $(2, 3)$ . Find the absolute maximum and absolute minimum for  $f$  on the domain  $D = \{(x, y) : 0 \leq x \leq 4, 0 \leq y \leq 5\}$

- (18) 6. Use Calc-3 techniques to find the maximum and minimum values for  $x^2 - y^2$  on the circle  $x^2 + y^2 = 1$