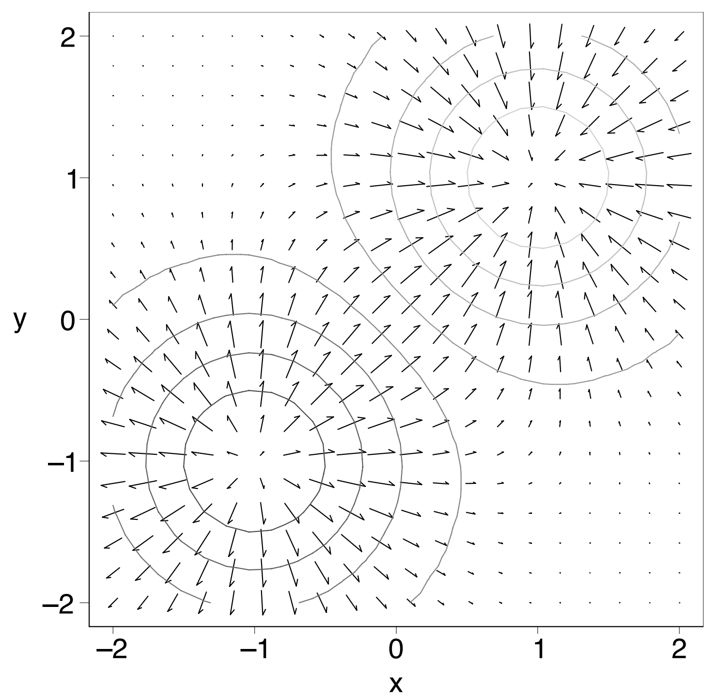
Math 275-003
March 10, 2006

Exam 2 Name ______________________________

This test consists of 100 points and 5 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

(10) 1. The following graph is a combined contour plot and gradient plot of a function of two variables.



The plot shows two critical values. Identify them as maxima or minima and explain why.

2. Let $f(x,y) = x\ln(x^2+y^2)$.

(5) (a) Find $\dfrac{\partial f}{\partial x}$.

(5) (b) Find $\dfrac{\partial f}{\partial y}$.

(5) (c) Find $\dfrac{\partial^2 f}{\partial y^2}$.

(5) (d) If $x = 2uv$ and $y = u^2 - v^2$, find $\dfrac{\partial f}{\partial u}$

(5) (e) If the unit vector $\mathbf{u} = \langle \frac{2}{\sqrt{13}}, \frac{3}{\sqrt{13}} \rangle$, find $D_{\mathbf{u}}(f)(1,1)$

(15) 3. Find equations of the tangent plane and the normal line to the surface $x^2 + 2y^2 + 3z^2 = 21$ at the point $(4, 1, -1)$

(10) 4. Show the following limit does not exist

$$\lim_{(x,y)\to(0,0)} \frac{2xy}{2x^2 + 3y^2}$$

(10) 5. The critical values of $f(x, y) = x^2 - 6xy + y^3$ are $(0, 0)$ and $(1, \frac{1}{2})$. Determine if f has a maximum, minimum or a saddle point at these two points.

(10) 6. Find the linear approximation of the function $f(x, y, z) = x^3\sqrt{y^2 + z^2}$ at the point $(2, 3, 4)$ and use it to approximate the number $(1.98)^3\sqrt{3.01^2 + 3.97^2}$

7. Let $f(x,y,z) = x^2 + 2y^2 + 3z^2$, let $g(x,y,z) = x + y + z$ and let $h(x,y,z) = x - y + 2z$

(10) (a) Set up (but do not solve) the equations needed to find the maximum and minimum values of $f(x,y,z)$ subject to the constraints $g(x,y,z) = 1$ and $h(x,y,z) = 2$.

(10) (b) The solution to the correct system of equations needed in part a is

$$\left\{ y = -\frac{6}{23}, \mu = \frac{30}{23}, \lambda = \frac{6}{23}, z = \frac{11}{23}, x = \frac{18}{23} \right\}$$

What are the maximum and minimum values of $f(x,y,z)$ subject to these constraints? Explain.