

This test consists of 4 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

- (10) 1. Show that the following limit does not exist.

$$\lim_{(x,y) \rightarrow (0,0)} \frac{2xy}{x^2 + y^2}$$

- (10) 2. Find the first partial derivatives of

$$f(x, y) = \sqrt{x^2 + 2y}$$

- (10) 3. Find an equation of the tangent plane to the surface $z = 3x^2 - y^2 + 2x$ at the point $(1, -2, 1)$
- (10) 4. Find the directional derivative of $f(x, y) = 2\sqrt{y} - x^2$ at the point $(1, 5)$ in the direction toward the point $(4, 1)$.
- (10) 5. What is the maximum rate of change of $f(x, y) = x^2y + \sqrt{y}$ at the point $(2, 1)$?

- (10) 6. Find the linear approximation of the function $f(x, y, z) = x^3 \sqrt{y^2 + z^2}$ at the point $(2, 3, 4)$ and use it to approximate the number $(1.98)^3 \sqrt{(3.01)^2 + (3.97)^2}$.

- (15) 7. Use Lagrange multipliers to find the maximum and minimum values for $f(x, y) = x^2 y$ subject to the condition that $x^2 + y^2 = 1$.

8. Let $f(x, y) = x^2 - xy + y^2 + 9x - 6y + 10$.

(10) (a) Find f_x , f_y , f_{xx} , f_{xy} , f_{yy} .

(10) (b) Find all the critical values for the function.

(5) (c) Identify the all the local maxima, minima and saddle points for the function.