

This test consists of 5 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

- (6) 1. Show that the following limit does not exist:

$$\lim_{(x,y) \rightarrow (0,0)} \left( \frac{x^2 + y^2}{x^2 + xy + y^2} \right)$$

2. The values of a function  $f$  are given in the table below with the first row being the  $x$  coordinates and the first column being the  $y$  coordinates:

	-20	-10	0	10	20
-20	2.4	2.1	0.8	0.5	1.0
-10	2.6	2.2	1.4	1.0	1.2
0	2.7	2.4	2.0	1.6	1.2
10	2.9	2.5	2.6	2.2	1.8
20	3.1	2.7	3.0	2.9	2.7

- (6) (a) Estimate:  $\frac{\partial f}{\partial x}(0,0)$

- (6) (b) Estimate:  $\frac{\partial f}{\partial y}(10,0)$

- (6) (c) Estimate the directional derivative  $D_{\vec{u}}(f)(0,10)$  if  $\vec{u} = \langle \frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2} \rangle$

- (6) 3. Use a linear approximation based at  $(-4, 3)$  to find a value of  $f(-4.01, 2.99)$  if  $f(x, y) = \sqrt{x^2 + y^2}$

- (6) 4. Find an equation of the tangent plane to the level surface defined by the equation

$$x^2z - y^2x + 3x - z = 4$$

at the point  $(1, -1, 2)$

- (12) 5. If  $z = \cos(xy) + y \cos(x)$  and  $x = u^2 + v$  and  $y = u - v^2$ , use the chain rule to find  $\partial z / \partial u$  and  $\partial z / \partial v$ .

- (12) 6. If  $yz^4 + x^2z^3 = e^{xyz}$  implicitly defines  $z$  as a function of  $x$  and  $y$ , find  $\partial z/\partial x$  and  $\partial z/\partial y$ .
- (6) 7. If  $f(x, y) = x^2 + xy^2$ , find the directional derivative of  $f$  at the point  $(2, 1)$  with  $\vec{u}$  parallel to  $\langle 3, 2 \rangle$ .
- (6) 8. Suppose that the elevation on a hill is given by  $f(x, y) = 100 - 4x^2 - 2y$ . From the site at  $(2, 1)$ , in which direction will the rain water flow? (The answer "Down hill" will not be awarded any points!) A solution in terms of a vector is acceptable.

9. Let  $f(x, y) = x^2 - xy + y^2 + 9x - 6y + 10$ .

(9) (a) Find and identify all local extrema for  $f$

(9) (b) Find the absolute extrema for  $f$  on the the region bounded by  $y = 0$ ,  $y = x$ , and  $x = 2$

(10) 10. Match the functions to the surfaces:

(a)  $f(x, y) = \frac{4}{2x^2+3y^2-1}$

(b)  $f(x, y) = x \sin(y)$

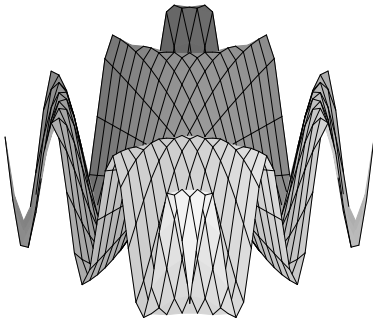
(c)  $f(x, y) = \sin(\sqrt{x^2 + y^2})$

(d)  $f(x, y) = \sin(xy)$

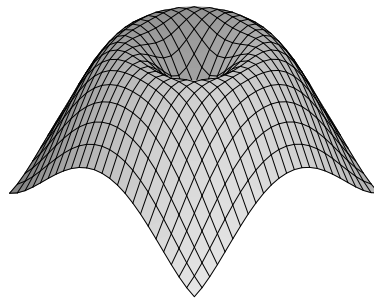
(e)  $f(x, y) = \frac{4}{2x^2+3x^2+1}$

(f)  $f(x, y) = \sin(\frac{x}{y})$

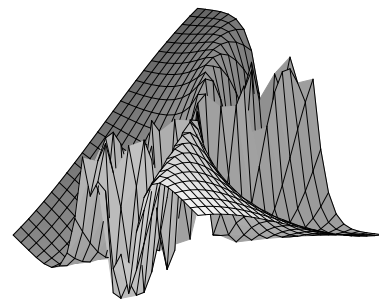
Surface A



Surface B

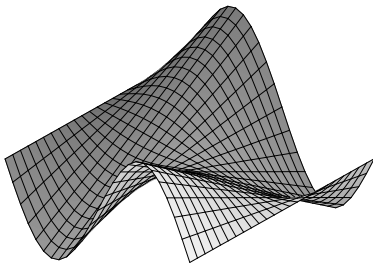


Surface C



Surface F

Surface D



Surface E

