Math 275 **Exam 2** Name ____________________

March 6, 2002

This test consists of 6 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

(10) 1. Draw a contour map for $f(x,y) = \dfrac{x-y}{x+y}$, showing the level curves $f(x,y) = -3$, $f(x,y) = -2$, $f(x,y) = -1$, $f(x,y) = 0$, $f(x,y) = 1$, $f(x,y) = 2$, $f(x,y) = 3$. From this contour map, what can you conclude about $\lim_{(x,y)\to(0,0)} f(x,y)$

2. Let $z = f(x, y) = y^2 - 3xy$.

(6) (a) Suppose $x = 2\cos(t)$ and $y = \sin(t)$. Use the chain rule to find dz/dt when $t = 0$.

(6) (b) Find $\nabla f(1, 2)$

(6) (c) Suppose $\vec{u} = \frac{\sqrt{5}}{5}\vec{\imath} + \frac{2\sqrt{5}}{5}\vec{\jmath}$. Find $D_{\vec{u}}f(1, 2)$, the directional derivative of f in the direction of $\vec{u}$.

(6) (d) Find the equation of the plane tangent to the graph of $z = f(x, y)$ at the point $(1, 2, -2)$.

(6) (e) Show that the only point where both $\frac{\partial f}{\partial x} = 0$ and $\frac{\partial f}{\partial y} = 0$ is $(0, 0)$.

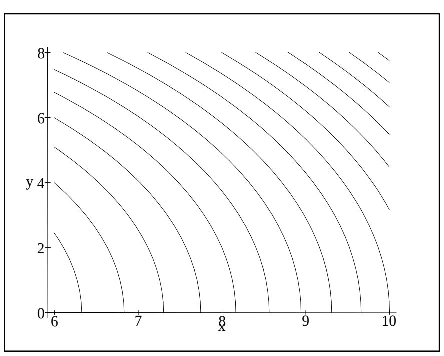
3. The depth of a lake at the point (x, y) is given by
$f(x, y) = 400 - 3x^2 - 2y^2$. A boy, who is not a very good swimmer, is in the water at $(8, 2)$.

(5) (a) Find the rate at which the depth changes if he swims in the direction of $\vec{i}$.

(5) (b) Find the rate at which the depth changes if he swims toward the point $(7, 4)$.

(5) (c) In what direction should he swim in order for the depth of the water to decrease most rapidly?

(5) (d) On the level curves given below, sketch the path the boy swims, starting at $(8, 2)$, if he always swims in the direction where the depth of the water decreases most rapidly.



(6) 4. Find a unit vector that is perpendicular to the graph of the hyperbola

$$\frac{x^2}{9} - \frac{y^2}{16} = 1$$

at the point $(9, 8\sqrt{2})$

(10) 5. Let $f(x, y) = \sin(\pi x + 2\pi y)$. Compute the tangent plane to the graph of f at the point $(0.5, 0.75)$, and use this plane to approximate $f(0.6, 0.8)$.

6. A metal plate is situated in the xy-plane and occupies the rectangle $0 \leq x \leq 10$ and $0 \leq y \leq 8$ where x and y are measured in meters. The temperature at the point (x, y) in the plane is $T(x, y)$ where T is measured in degrees celsius. Temperatures at equally spaces points were measured and recorded in the table

$x \setminus$ y	0	2	4	6	8
0	30	38	45	51	55
2	52	56	60	62	61
4	78	74	72	68	66
6	98	87	80	75	71
8	96	90	86	80	75
10	92	92	91	87	78

(5) (a) Estimate the values of the partial derivatives $T_x(6, 4)$ and $T_y(6, 4)$.

(5) (b) Estimate the value of $D_{\vec{u}}T(6, 4)$ where $\vec{u} = (\vec{i} + \vec{j})/\sqrt{2}$.

(10) 7. Find the points on the sphere $x^2 + y^2 + z^2 = 1$ where the tangent plane is parallel to the plane $2x + y - 3z = 2$

(4) 8. Do you know your name?