M272/275–001
October 6, 1998

TEST #2 Name _____

This test consists of **4** pages, all different and none intentionally left blank.
Take a minute *right now* to ensure that you have all 4 of these pages.
In order to receive credit for your answers, you must show your work!!

1. (10 points) Suppose a particle has position function $\vec{r}(t) = \langle t\cos(t), t\sin(t), t^2\rangle$ Find the tangential and normal components of the acceleration vector for this particle.

2. (10 points) Find the curvature of the ellipse $\mathbf{r}(t) = 2\cos(t)\mathbf{i} + 3\sin(t)\mathbf{j} + (\sin(t) + \cos(t))\mathbf{k}$ at the point $(2, 0, 1)$ and at the point $(0, -3, -1)$

3. (10 points) Sketch the solid consisting of all points with spherical coordinates (ρ, θ, ϕ) such that $0 \leq \theta \leq \pi/2$, $0 \leq \phi \leq \pi/6$ and $0 \leq \rho \leq 2\cos(\phi)$

4. (10 points) Prove that if a particle moves at constant speed, then the acceleration vector is perpendicular to the velocity vector.

5. Let $z = y^2 - 5x^2y$. (10 points each part)

a. Suppose $x = 2\sin(t+\pi)$ and $y = \cos(t)$. Use the chain rule to find dz/dt when $t = 0$.

b. Find $\nabla f(1,2)$

c. Suppose $\vec{u} = \frac{\sqrt{5}}{5}\vec{i} + \frac{2\sqrt{5}}{5}\vec{j}$. Find $D_{\vec{u}}f(1,2)$, the directional derivative of f in the direction of $\vec{u}$.

d. Find the equation of the plane tangent to the graph of $z = f(x,y)$ at the point $(1,2,-9)$.

6. You are given only the following information about a function f:

$$f(8,5) = 25.2 \qquad f(8.01,5) = 25.0 \qquad f(8,5.02) = 25.8$$

(a) (5 points) Approximate $\dfrac{\partial f}{\partial x}(8,5)$ and $\dfrac{\partial f}{\partial y}(8,5)$

(b) (8 points) Approximate the equation of the tangent plane to the surface at (8.5).

(c) (7 points) Using the tangent plane, approximate $f(8.01, 5.02)$