Math 275-030
June 18, 2009

# Exam 1

This test consists of 100 points and 9 questions. Take a few seconds right now to be sure you have all the questions. The point value of each question is to the left of the question number. Show all your work in the bluebook as specified. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

1. (Page 1) Let $\mathbf{u} = \mathbf{i} + \mathbf{j}$ and $\mathbf{v} = 2\mathbf{i} + \mathbf{j} - 2\mathbf{k}$. Find each of the following

(3) (a) $3\mathbf{u} + 2\mathbf{v}$

(3) (b) $|\mathbf{v}|$

(3) (c) $|\mathbf{u}|$

(3) (d) $\mathbf{u} \cdot \mathbf{v}$

(3) (e) $\mathbf{v} \cdot \mathbf{u}$

(3) (f) $\mathbf{u} \times \mathbf{v}$

(3) (g) $\mathbf{v} \times \mathbf{u}$

(3) (h) the angle between $\mathbf{u}$ and $\mathbf{v}$

(3) (i) the scalar component of $\mathbf{u}$ in the direction of $\mathbf{v}$

(3) (j) The vector projection of $\mathbf{u}$ onto $\mathbf{v}$

(5) 2. (Top third of page 2) Find the distance from the point $(1, 2, 3)$ to the line $x = 3 - t, \quad y = 1 + 2t \;\; z = 5 - t$

(5) 3. (Middle third of page 2) Find the point in which the line through the origin $(0, 0, 0)$ perpendicular to the plane $2x - y - z = 4$ meets the plane $3x - 5y + 2z = 6$

(5) 4. (Bottom third of page 2) Find the distance from the point $(2, 2, 3)$ to the plane $2x+3y+5z = 0$

(10) 5. (Page 3) Identify the following quadric surface and sketch its graph.

$$x^2 + 4\,x + 8 - y^2 + 2\,y + z^2 + 6\,z = 0$$

(10) 6. (Page 4) At what times in the interval $0 \leq t \leq \pi$ are the velocity and acceleration vectors of the motion $\mathbf{r}(t) = (t + \sin(t))\mathbf{i} + (1 + \cos(t))\mathbf{j}$ orthogonal?

(15) 7. (Page 5) Find the tangent vector $\mathbf{T}$, the normal vector $\mathbf{N}$, and the curvature $\kappa$ for the space curve

$$\mathbf{r}(t) = \left\langle \frac{t^3}{3}, t^2, 2t \right\rangle$$

(10) 8. (Page 6) A particle starts at the origin with initial velocity $\mathbf{i} - \mathbf{j} + 3\mathbf{k}$. It's acceleration is $\mathbf{a}(t) = 6t\,\mathbf{i} + 12t^2\,\mathbf{j} - 6t\,\mathbf{k}$. Find its position function.

(10) 9. (Page 7) Let $y = f(x)$ be a twice differentiable function. The graph of the curve $\mathbf{r}(t) = t\mathbf{i} + f(t)\,\mathbf{j}$ is the same as the graph of the function $y = f(x)$. Use this fact to show that the curvature of $\mathbf{r}(t)$ is zero at any inflection point of $f$.