Math 275-001 **Exam 1** Name _____
February 19, 2008

This test consists of 100 points and 5 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

1. Let $\mathbf{a} = \langle 3, 1, 5 \rangle$ and $\mathbf{b} = \langle -2, 4, 2 \rangle$

(3) (a) Find $\mathbf{a} \cdot \mathbf{b}$.

(3) (b) Find $\mathbf{a} \times \mathbf{b}$.

(3) (c) Find the cosine of the angle between $\mathbf{a}$ and $\mathbf{b}$

(3) (d) Find the scalar projection of $\mathbf{a}$ onto $\mathbf{b}$

(8) 2. Use vector concepts to find the area of the triangle whose vertices are at the points $(-1, 2, 1)$, $(5, 2, 3)$ and $(3, 4, -2)$

(7) 3. Find an equation of the line through the point $(-2, 3, 1)$ that is parallel to the vector $< 2, -4, 3 >$

4. This problem refers to the lines

$$L_1 : x = 3 + 2t, y = 4 - 3t, z = -2 - t$$

and

$$L_2 : x = 1 - 6t, y = 7 + 2t, z = -1 + 3t$$

(7) (a) Find the point of intersection of the lines

(6) (b) Find an equation of the plane containing these lines.

(10) 5. The following quadric surface in in standard form, identify the surface and sketch its graph.

$$\frac{(x-1)^2}{4} + \frac{(y-2)^2}{9} - \frac{(z+3)^2}{4} = 1$$

(10) 6. Sketch the solid consisting of all points with spherical coordinates (ρ, θ, ϕ) such that $0 \leq \theta \leq \pi/2$, $0 \leq \phi \leq \pi/6$ and $0 \leq \rho \leq 2\cos(\phi)$.

(10) 7. The helix $\mathbf{r}_1(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j} + t\mathbf{k}$ intersects the curve $\mathbf{r}_2(t) = (1+t)\mathbf{i} + t^2\mathbf{j} + t^3\mathbf{k}$ at the point $(1, 0, 0)$ Find the cosine of the angle of intersection of these curves.

(10) 8. Set up (but do not evaluate) an integral whose value is the length of the graph of $\mathbf{r}(t) = \langle t, t^2, \ln(t)\rangle$ from the point $(1, 1, 0)$ to the point $(3, 9, \ln(3))$

9. The position vector of a particle is given by $\mathbf{r}(t) = \langle t^2, 5t, t^2 - 16t \rangle$.

(5) (a) Find the velocity vector for this function at $t = 4$.

(5) (b) Find the acceleration vector for this function at $t = 4$

(5) (c) Find the speed for this function at $t = 4$

(5) (d) Find the curvature at $t = 4$.