Math 275-002 Exam 1 Name _____
September 23, 2008

This test consists of 100 points and 4 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

No Graphing Calculators

1. Let $\mathbf{u} = \langle 2, -1, 5 \rangle$ and $\mathbf{v} = \langle -1, 3, -3 \rangle$.

(8) (a) Find the dot product $\mathbf{u} \cdot \mathbf{v}$

(8) (b) Find the cross product $\mathbf{v} \times \mathbf{u}$

(8) (c) Find the cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$

(8) (d) Find the vector projection of $\mathbf{u}$ onto $\mathbf{v}$

(10) 2. Let $P(1, 2, 4)$ and $Q(3, 2, 1)$ be points in $\mathbb{R}^3$. What are the coordinates of the point on the line through P and Q which is $2/3$ of the way from P to Q.

3. Let L_1 be the line given by the vector equation $\mathbf{r}_1(t) = \langle 3, 1, 4\rangle + t\langle 2, -1, 1\rangle$ and let L_2 be the line given by the vector equation $\mathbf{r}_2(t) = \langle 5, 7, 2\rangle + t\langle 8, 10, -2\rangle$.

(8) (a) Show that these lines intersect and find the point of intersection.

(8) (b) Find an equation of the plane which contains these two lines.

4. A person on a hang glider is spiraling upward due to rapidly rising air on a path having position vector $\mathbf{r}(t) = 2\cos(t)\mathbf{i} + 2\sin(t)\mathbf{j} + t^2\mathbf{k}$

(8) (a) Find the velocity and acceleration vectors

(8) (b) Find the glider's speed

(8) (c) Find the tangential and normal components of the acceleration.

(8) (d) Find the curvature κ at time $t = \pi$

(10) 5. Find the length of the curve given by the vector valued function

$$\mathbf{r}(t) = \langle t\cos(t), t\sin(t), \frac{2\sqrt{2}}{3}t^{3/2}\rangle$$

from $t = 0$ to $t = 2\pi$