Math 275-030 **Exam 1** Name ____________________
June 14, 2007

This test consists of 100 points and 5 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

1. Let P and Q be points in $\mathbb{R}^3$ having coordinates $(2, 5, 4)$ and $(-2, 4, 3)$ respectively. Let $\vec{u}$ be the vector from the origin to P and let $\vec{w}$ be the vector from the origin to Q.

(6) (a) Compute $\vec{u} \cdot \vec{w}$

(6) (b) Compute $\vec{u} \times \vec{w}$

(6) (c) Find parametric equations for the line through the points P and Q

(6) (d) Find an equation of the plane through P that is perpendicular to the line through the points P and Q.

(6) (e) Find the projection of $\vec{u}$ onto $\vec{w}$.

(6) (f) What is the angle between $\vec{u}$ and $\vec{w}$ (You may leave your answer in terms of an inverse trig function if you wish.)

2. Let Ψ be the plane that intersects the x axis at 3, the y axis at -2 and the z axis at 4.

(6) (a) What is an equation of this plane?

(6) (b) Find a normal vector, $\vec{n}$, to this plane.

(6) (c) Let P be the point $(6, 4, 6)$ and let $\vec{u}$ be the vector from $(0, -2, 0)$ to P. Find the component of $\vec{u}$ onto $\vec{n}$ (from the previous part.)

(6) (d) What is the distance from P to Ψ?

(10) 3. Suppose the lines L_1 and L_2 and given by the vector equations $\vec{r}_1(t) = \langle 1+2t, 3-t, 2+3t\rangle$ and $\vec{r}_2(t) = \langle 1+3t, 8+t, -1+3t\rangle$. Do these lines intersect? If so, what is the point of intersection of the lines?

(10) 4. Find the equation of the line of intersection of the planes

$$2x + 3y - z = 6$$
$$x + y + z = 9$$

5. Let $\vec{r}(t) = \langle 2\cos(t),\ \ 3\sin(t),\ \ t+1 \rangle$.

(5) (a) Set up an integral whose value is the length of the graph of $\vec{r}$ from $t = 1$ to $t = 4$.

(5) (b) Find the curvature of the graph of $\vec{r}$ when $t = \pi$.

6. For the curve given by $\vec{r}(t) = \langle \frac{1}{3}t^3, \;\; \frac{1}{2}t^2, \;\; t \rangle$, find

(5) (a) The unit tangent vector.

(5) (b) The unit normal vector.