Math 275-003 Exam 1 Name _____
February 13, 2006

This test consists of 100 points and 5 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

1. Let $\mathbf{v} = \langle 3, -2, 5 \rangle$ and $\mathbf{u} = \langle 4, -2, 4 \rangle$.

(6) (a) Find $|\mathbf{v}|$.

(6) (b) Find $\mathbf{v} \cdot \mathbf{u}$.

(6) (c) Find $\mathbf{u} \times \mathbf{v}$.

(6) (d) Find the cosine of the angle between $\mathbf{u}$ and $\mathbf{v}$.

(6) (e) Find $\text{comp}_{\mathbf{u}}(\mathbf{v})$

(6) 2. (a) Find parametric equations for the line through the points $(2, -3, 2)$ and $(4, 2, -4)$

(6) (b) Do the lines $\mathbf{r}_1(t) = \langle -4, -4, 4\rangle + t\langle 5, 6, -1\rangle$ and $\mathbf{r}_2(t) = \langle -5, 4, 11\rangle + t\langle -3, 1, 4\rangle$ intersect? If so, where?

(6) (c) Find parametric equations of the line through $(1, 2, 3)$ which is perpendicular to the plane given by $-x + 2y - 3z = 6$

(6) (d) Find an equation of the plane through the points $(2, 0, 0)$, $(0, -1, 0)$ and $(0, 0, 5)$.

(10) 3. Reduce the following equation to one of the standard forms, classify the surface and sketch it.

$$4y^2 + z^2 - x - 16y - 4z + 20 = 0$$

(10) 4. A solid in $\mathbb{R}^3$ lies above the cone $z = \sqrt{x^2 + y^2}$ and inside the sphere $x^2 + y^2 + z^2 = 5$ Write a description of the solid in terms of inequalities involving spherical coordinates.

(10) 5. The vector valued function $\mathbf{r}(t) = \langle t^2, 2t, \ln(t) \rangle$ for $1 \leq t \leq e$ traces a curve segment in $\mathbf{R}^3$. Set up an integral whose value is the length of this segment. DO NOT EVALUATE THIS INTEGRAL!

6. The position of a particle in $\mathbb{R}^3$ at time t is given by the vector equation

$$\mathbf{r}(t) = \cos(t)\mathbf{i} + \sin(t)\mathbf{j} + t\mathbf{k}$$

(6) (a) Find the tangent vector $\mathbf{T}$ at the point $\mathbf{r}(\pi/2)$.

(5) (b) Find the normal vector $\mathbf{N}$ and the binormal vector $\mathbf{B}$ at the point $\mathbf{r}(\pi/2)$.

(5) (c) The plane determined by the normal $\mathbf{N}$ and the binormal $\mathbf{B}$ at the point P is called the normal plane at the point P. Find an equation of the the normal plane at the point $\mathbf{r}(\pi/2)$.