Math 275 Exam 1 Name _____
February 13, 2002

This test consists of 5 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

(10) 1. Find an equation of the sphere whose center is at the point $(-3, 5, -5)$ and whose graph is tangent to the xz-plane.

(10) 2. The graph of the following quadratic equation is one of the standard quadratic surfaces. Identify the name of the surface and sketch its graph. (You might want to complete the square first.)

$$z^2 = 4x^2 + y^2 + 8x - 2y + 4z$$

(10) 3. Sketch the solid in 3-space that is described in spherical coordinates by the following inequalities:

$$0 \leq \rho \leq 2 \text{ and } \pi/6 \leq \phi \leq \pi/3$$

(10) 4. Find parametric equations of the line of intersection of the planes

$$2x + y - 3z = 3 \text{ and } x + 2y + 3z = 3$$

(10) 5. Sketch the graph of $\vec{r}(t) = (1 + \cos(t))\vec{i} + (3 - \sin(t)\vec{j}$ and show the direction of increasing t. (Hint: $\sin^2(t) + \cos^2(t) = 1$ should help you figure out the curve.)

(10) 6. Use the fact that the curvature is given by the formula

$$\kappa(t) = \frac{|\ \vec{r}\,'(t) \times \vec{r}\,''(t)\ |}{|\ \vec{r}\,'(t)\ |^3}$$

to find the curvature of

$$\vec{r}(t) = \langle a\cos(t), a\sin(t), ct\rangle$$

at the point $t = \pi/2$.

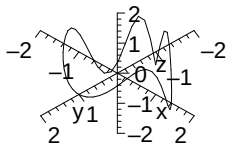
(10) 7. Find an equation of the plane passing through the three points $(5,0,0)$, $(0,-3,0)$ and $(0,0,2)$.

(12) 8. Match the following vector valued function (a-d) with its graph (I-IV). Give reasons for your answers.

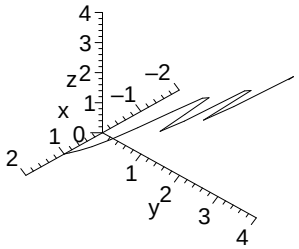
a. $\vec{r}(t) = \langle \cos(t^2), t, t \rangle$ b. $\vec{r}(t) = \langle \cos(t), \sin(t), \sin(t^2) \rangle$

c. $\vec{r}(t) = \langle \sin(t^2), \cos(t^2), t \rangle$ d. $\vec{r}(t) = \langle t, t, 6 - t^2 \rangle$

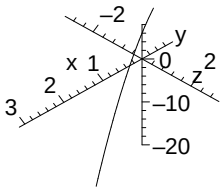
Graph I



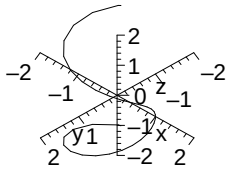
Graph II



Graph III



Graph IV



(10) 9. If $\vec{r}(t)$ has constant length, prove that $\vec{r}\,'$ is orthogonal to $\vec{r}$

(8) 10. Find the length of the curve traced out by $\vec{r}(t) = 2t\vec{i} + \ln(t)\vec{j} + t^2\vec{k}$ as t goes from 1 to e.