M272/275
September 14, 1998 TEST #1 Name _____

This test consists of **4** pages, all different and none intentionally left blank.
Take a minute *right now* to ensure that you have all 4 of these pages.
In order to receive credit for your answers, you must show your work!!

1. (10 points each) Let $\mathbf{v} = \langle 3, 2, -4 \rangle$ and $\mathbf{u} = \langle -2, -4, 1 \rangle$ be vectors in $\mathbb{R}^3$. Compute each of the following quantities

   a. $\mathbf{v} \cdot \mathbf{u}$

   b. $\mathbf{v} \times \mathbf{u}$

   c. Find the angle between $\mathbf{u}$ and $\mathbf{v}$

   d. Find symmetric equations for the line through the point $(3, -2, 4)$ parallel to the vector $\mathbf{v}$.

   e. Find an equation of the plane containing the vectors $\mathbf{u}$ and $\mathbf{v}$ and the point $(2, 3, -1)$

2. (10 points) Find the length of the curve $\mathbf{r}(t) = \langle 2t^{3/2}, \sin(2t), \cos(2t) \rangle \quad 0 \leq t \leq 1$

3. (10 points) A curve is given by parametric equations $x = t, \quad y = t^2, \quad z = \frac{4}{3}t^{3/2}$, with $t \geq 0$. Find the unit tangent vector and the unit normal vector for the curve at the point $(1, 1, \frac{4}{3})$

4. (10 points) Identify each of the following statements as *True* or *False* (I assume you know that this means to label the true statements with a *true* and the false ones with a *false*)

   _____ a. For any pair of vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$, $\mathbf{u} \cdot \mathbf{v} = \mathbf{v} \cdot \mathbf{u}$

   _____ b. For any pair of vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$, $\mathbf{u} \times \mathbf{v} = \mathbf{v} \times \mathbf{u}$

   _____ c. For any pair of vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$ and any scalar k, $(k\mathbf{u}) \cdot \mathbf{v} = k(\mathbf{u} \cdot \mathbf{v})$

   _____ d. For any pair of vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$, $(k\mathbf{u}) \times \mathbf{v} = k(\mathbf{u} \times \mathbf{v})$

   _____ e. For any vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w} \in \mathbb{R}^3$, $(\mathbf{u} + \mathbf{v}) \times \mathbf{w} = \mathbf{u} \times \mathbf{w} + \mathbf{v} \times \mathbf{w}$

   _____ f. For any vectors $\mathbf{u}$, $\mathbf{v}$ and $\mathbf{w} \in \mathbb{R}^3$, $\mathbf{u} \cdot (\mathbf{v} \times \mathbf{w}) = (\mathbf{u} \times \mathbf{v}) \cdot \mathbf{w}$

   _____ g. For any pair of vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$, $(\mathbf{u} \times \mathbf{v}) \cdot \mathbf{u} = 0$

   _____ h. For any pair of vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$, $(\mathbf{u} + \mathbf{v}) \times \mathbf{v} = \mathbf{u} \times \mathbf{v}$

   _____ i. The set of points that satisfies the linear equation $Ax+By+Cz = D$ represents a line in space.

   _____ j. The set of points $\{(x, y, z) | x^2 + y^2 = 1\}$ is a circle.

5. (10 points) Show that the lines with parametric equations

   $$x = 3 + t, \quad y = -1 + t, \quad z = 2 - 3t$$

   and

   $$x = 3 + t, \quad y = 1 + 2t, \quad z = -1 + 3t$$

   are skew lines. Then find the distance between these lines.

6. (10 points) Identify and sketch the graph of

$$y^2 + 4x^2 - 9z^2 = 36$$