M204
December 12, 1989

FINAL Name _ *Find the derivative of each of the functions given in problems 1–6. Simplify your answers for problems 1–3.*

1. $f(x) = 2x^2 - 5x + 4$
2. $f(x) = (x+4)^3(x-3)^2$
3. $f(x) = \dfrac{x}{\sqrt{x^2+1}}$
4. $f(x) = \int_0^x \sqrt{1+t^4}\, dt$
5. $f(x) = \dfrac{2x^2 \ln(\sin(x)) + 5}{\arctan(x) + 1}$
6. $f(x) = (x + (2x + (3x+4)^5)^6)^7$

Evaluate the following limits (Problems 7 – 11):

7. $\lim_{x\to 2}(x^2 + 5x - 2)$
8. $\lim_{x\to 2} \dfrac{x^2 - 6x + 8}{x - 2}$
9. $\lim_{x\to\infty} \left(\dfrac{x+2}{x}\right)^x$
10. $\lim_{x\to 0} \sin(x)\ln(\tan(x))$
11. $\lim_{x\to -\infty} (x + \sqrt{x^2 - 2x})$

Evaluate each of the integrals in problems 12 – 16.

12. $\int (2x^2 - x + 4 - \dfrac{x}{1+x^2})\, dx$
13. $\int_0^2 (x\sqrt{4-x^2})\, dx$
14. $\int \dfrac{e^{\tan(x)}}{\cos^2(x)}\, dx$
15. $\int \dfrac{1}{x^2 + 2x + 5}\, dx$
16. $\int \dfrac{e^x + e^{-x}}{e^x - e^{-x}}\, dx$
17. Find (and identify) the local extrema of the function $f(x) = (x-2)^2(x-4)^3$.
18. Give a calculus proof to show that among all rectangles of area 9 square feet, the square has the smallest perimeter.

19. State the ϵ-δ definition of the *limit* as x goes to a of $f(x)$.

20. State the *Fundamental Theorem of Calculus.*

21. Using the (*limit*) definition of the derivative, prove that the slope of the tangent line to the graph of $f(x) = x^2$ at the point $(3, 9)$ is 6.

22. Give an "ϵ-δ" proof of the fact that $\lim_{x \to 3}(4 - x) = 1$.

23. Sketch the graph of

$$f(x) = \frac{x^2}{(x-2)^2}.$$

24. If the length of a side of an equilateral triangle is increasing at a rate of 2 in./min, how fast is the area of the triangle increasing when the side is 7 inches long.

25. Find the area of the region between the graphs of $y = \cos(x)$ and $y = 2$ from $x = \pi$ to $x = 2\pi$.