M170–004
April 24, 2000

TEST #4 Name _____

This test consists of **5** pages, all different and none intentionally left blank.
Take a minute *right now* to ensure that you have all 5 of these pages.
In order to receive credit for your answers, you must show your work!!

1. (11 points) Find the maximum and minimum values for

$$f(x) = 1 - 3x + 5x^2 - x^3$$

on the interval $[-2..4]$

2. (a) (5 points) State the Mean-Value Theorem.

(b) (6 points) Draw a picture of a nonlinear function which satisfies the hypotheses of the Mean Value Theorem and illustrate on this graph a geometric interpretation of the Mean Value Theorem.

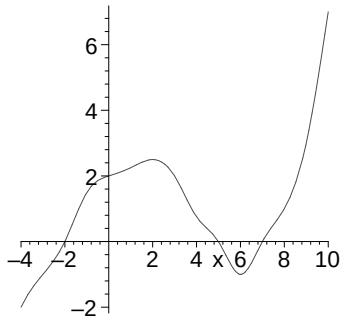
3. (11 points) Sketch the graph of a function that satisfies all of the following conditions.
   $f(0) = 0, \quad f(-1) = 1; \quad f'(-1) = 0$
   $f''(x) > 0$ on $(-\infty, -1)$
   $f''(x) < 0$ on $(-1, 0)$ and on $(0, \infty)$
   $f'(x) > 0$ for $x > 0$ $f' < 0$ for $x < 0$

4. (11 points) Sketch a graph of the function f defined below. To help, I have given you the simplified formulae for the first and second derivatives of f as well as the formula of f itself. Since you can plot f using your calculators, the actual plot does not count very many points (not more than 2.) The points are awarded based upon your work.

$$\begin{aligned} f(x) &= \frac{x+2}{(1-x)^{3/2}} \\ f'(x) &= -\frac{x+8}{2(x-1)^{5/2}} \\ f''(x) &= \frac{3(x+14)}{4(x-1)^{7/2}} \end{aligned}$$

5. (11 points) If $f'(x) < 1$ for all $x \in [-4, 4]$ and $f(-1) = 0$, is it possible for $f(2) = 4$? Why or why not?

6. Use the following graph of f' to answer the following questions about f. (N.B. The graph of f is not shown!!)



(a) (5 points) On what intervals is f increasing and on what intervals is f decreasing?

(b) (5 points) At what values of x does f have a local maximum and at what values of x does f have a local minimum.

(c) (5 points) On what intervals is the graph of f concave up and on which intervals is the graph of f concave down.

(d) (5 points) Assuming f is continuous, and that $f(0) = 0$, sketch a possible graph of f.

7. (10 points) Find the area of the largest rectangle that can be inscribed in the upper half of a circle of radius 6 if one side of the rectangle must be on a diameter of the circle.

8. (15 points) If $f(1) = 2$, $f'(-1) = 1$ and $f''(x) = -2x + 4$, what is the formula for f?