M170–003
April 21, 1999

TEST #4 Name _____

This test consists of **5** pages, all different and none intentionally left blank.
Take a minute *right now* to ensure that you have all 5 of these pages.
In order to receive credit for your answers, you must show your work!!

1. (12 points) The conclusion of the Mean Value theorem states $\exists c \in (a, b)$ so that

$$\frac{f(b) - f(a)}{b - a} = f'(c)$$

Draw a picture of a nonlinear function which satisfies the hypotheses of the Mean Value Theorem and illustrate a geometric interpretation of the Mean Value Theorem.

2. (12 points) Using calculus (and algebra), find the intervals on which the graph of $f(x) = x^4 - 2x^3 - 12x^2 + 2x + 4$ is concave up and those on which the graph of f is concave down. Identify the inflection points for f.

3. (12 points) Sketch a graph of the function f defined below. To help, I have given you the simplified formulae for the first and second derivatives of f as well as the formula of f itself. Since you can plot f using your calculators, the actual plot does not count very many points (not more than 2.) The points are awarded based upon your work.

$$\begin{aligned} f(x) &= \frac{x-2}{(x-1)^{3/2}} \\ f'(x) &= -\frac{x-4}{2(x-1)^{5/2}} \\ f''(x) &= \frac{3(x-6)}{4(x-1)^{7/2}} \end{aligned}$$

4. (10 points) Sketch the graph of the function that has the following properties:

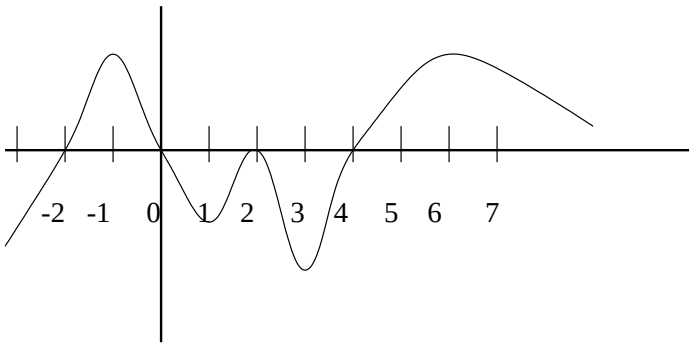
$$f(-3) = 2, \quad f(-1) = 5, \quad f(2) = -4, \quad f(6) = -1,$$

$$f'(-1) = f'(2) = 0, \quad f''(-3) = f''(6) = 0$$

$$\lim_{x \to -\infty} f(x) = 1, \quad \lim_{x \to 0^-} f(x) = -\infty, \quad \lim_{x \to 0^+} f(x) = +\infty, \quad \lim_{x \to \infty} f(x) = 0,$$

$$f''(x) > 0, \text{ if } x < -3 \text{ or } 0 < x < 6 \text{ and } f''(x) < 0, \text{ if } -3 < x < 0 \text{ or } 6 < x$$

5. (12 points) Using the graph of f', given below, determine the x coordinates of the local extrema and inflection points of f. Be sure to distinguish between the maxima and minima. (Note: The graph of f is not given).



6. (12 points) Find (and identify) the local extrema for $f(x) = x(x^2 - 16)^{1/3}$

7. (12 points) If $f(-2) = -3$ and $f(1) = 0$, is it possible for $f'(x) < 1$ for all $x \in [-2, 1]$? Justify your answer.

8. (12 points) A rectangular plot of ground is to be fenced on three sides to form a garden, with the fourth side bounded by a retaining wall. If there are 50 feet of fencing available, what should be the dimensions of the garden to produce the maximum area?