

M170-003

April 12, 1999

TEST #3 Name _____

This test consists of 4 pages, all different and none intentionally left blank.

Take a minute *right now* to ensure that you have all 4 of these pages. In order to receive credit for your answers, you must show your work!!

NO CALCULATORS

1. (5 points each) Find the derivative of each of the following functions.

(a) $f(x) = e^{2x^2+2x-5}$

(b) $f(x) = \log_5(\sec(x))$

(c) $g(x) = \arcsin(x/2)$

(d) $h(x) = \sinh(2x)$

2. (10 points) Let f be a one-to-one function. Suppose $f(5) = 2$ and $f'(5) = 3$. Find an equation of the tangent to the graph of $g(x) = f^{-1}(x)$ at $x = 2$.

3. (8 points each) Evaluate the following limits

(a) $\lim_{x \rightarrow 0} (1 - \cos(x))^{1/x}$

(b) $\lim_{x \rightarrow 0^+} \sqrt{x} \ln(x)$

(c) $\lim_{x \rightarrow 0} \frac{e^{2x} - 1}{e^x - 1}$

$$(d) \lim_{x \rightarrow \infty} \frac{\ln(1 + e^{2x})}{x}$$

$$(e) \lim_{x \rightarrow +\infty} (x + \sqrt{x^2 - 2x})$$

4. (10 points) The radius of a spherical ball bearing is measured to be 0.7 with a maximum error of ± 0.01 inches. Use differentials to approximate the error in the volume of the ball bearing. Remember, the volume of a sphere is given by $\frac{4}{3}\pi r^3$.

5. (10 points) Find the quadratic approximation to $f(x) = \sin(x)$ at $x = \frac{\pi}{3}$

6. (a) (5 points) Show that $\sin(\operatorname{arccot}(x)) = \frac{1}{\sqrt{x^2 + 1}}$

(b) (5 points) Now show that $\cos(\arctan(\sin(\operatorname{arccot}(x)))) = \sqrt{\frac{x^2 + 1}{x^2 + 2}}$