

This test consists of 100 points and 4 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

### NO GRAPHING CALCULATORS

(10) 1. Let

$$f(x) = 2x^3 - 3x^2 - 12x$$

on the interval  $I = [-3, 3]$ . Find the absolute maximum and minimum for  $f$  on  $I$ .

(10) 2. Let  $f(x) = \ln(x)$ . Explain why  $f$  satisfies the hypotheses of the Mean Value Theorem on  $[1, e]$  and find the value of  $c$  that satisfies the conclusion of the Mean Value Theorem.

- (10) 3. Let  $f(x) = 2x^3 - 3x^2 - 12x + 3$ . Find intervals on which the graph of  $f$  is increasing and those on which the graph of  $f$  is decreasing. (Note that this is essentially the function from problem 1.)
- (20) 4. A right circular cone is inscribed in a sphere of radius 4. Find the largest possible volume for such a cone. (The volume of a cone of radius  $r$  and height  $h$  is  $V = \frac{1}{3}\pi r^2 h$ .)

- (20) 5. This is your graphing problem. I have computed and simplified the first and second derivative. Determine where the graph is increasing and decreasing, the local extrema, the inflection points and the asymptotes for the given function.

$$f(x) = 4 \frac{x - 3}{(x - 1)^{3/2}}$$

$$f'(x) = -2 \frac{x - 7}{(x - 1)^{5/2}}$$

$$f''(x) = 3 \frac{x - 11}{(x - 1)^{7/2}}$$

6. Evaluate the following limits:

$$(10) \quad (a) \lim_{x \rightarrow 0} \frac{\cos(mx) - \cos(nx)}{x^2}$$

$$(10) \quad (b) \lim_{x \rightarrow 0^+} (\sin(x) \ln(x))$$

$$(10) \quad (c) \lim_{x \rightarrow \infty} \left(1 + \frac{a}{x}\right)^{bx}$$