

Math 170-007
November 17, 2004

Exam 3 Name _____

This test consists of 5 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

(10) 1. Find the maximum and minimum values of $f(x) = 10 + 27x - x^3$ on the interval $[0, 4]$.

(10) 2. Find all inflection points of $f(x) = x^4 + x^3 - 3x^2 + 2$

(15) 3. Sketch the graph of a function that satisfies the following conditions:

$$\begin{aligned}f(0) &= 0, \quad f'(-2) = f'(1) = f'(9) = 0, \\ \lim_{x \rightarrow \infty} f(x) &= 0, \quad \lim_{x \rightarrow 6} f(x) = -\infty \\ f'(x) &< 0 \text{ on } (-\infty, -2), (1, 6) \text{ and } (9, \infty) \\ f'(x) &> 0 \text{ on } (-2, 1) \text{ and } (6, 9) \\ f''(x) &> 0 \text{ on } (-\infty, 0) \text{ and } (12, \infty) \\ f''(x) &< 0 \text{ on } (0, 6) \text{ and } (6, 12)\end{aligned}$$

- (10) 4. Suppose $f'(x) < 3$ on the interval $[-2, 2]$ and that $f(2) = 9$. Use the mean value theorem to prove $f(-2) > -3$

- (10) 5. What is the value of $\lim_{x \rightarrow \infty} \frac{\ln(e^x + x)}{x}$?

(10) 6. If $f'(x) = 8x^3 + 12x + 3$ and $f(1) = 6$, what is the formula for $f(x)$?

7. Let $f(x) = \sqrt{2x+1} - \sqrt{x+4}$.

(7) (a) Show $f(x) = 0$ has a solution in the interval $[2, 4]$.

(8) (b) Starting with an initial estimate of $x = 3$ for this solution, use one iteration of Newton's method to compute the next estimate of the solution.

8. A triangle has vertices at the points $(0,12)$, $(-6,0)$ and $(6,0)$ in the cartesian plane. Another triangle is inscribed in this triangle with vertices $(0,0)$, $(-a,b)$ and (a,b) .
- (5) (a) Draw a picture showing these triangles and the coordinate axes.
- (5) (b) What is an equation that must be satisfied by a and b ?
- (5) (c) Using a and b , what is the area of the inscribed triangle.
- (5) (d) For what values of a and b will the area of the inscribed triangle be a maximum?