MATH 170
April 17, 2001

Exam 3 Name _____

This test consists of 5 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. When you finish the exam, you may hand in your paper and quietly leave.

(10) 1. Sketch the graph of a continuous function satisfying the following: Be sure to include your reasons for the shape of your graph. No credit for just a picture.

$f(x) < 0$ for $1 < x < 3$; $f(x) > 0$ for $x > 3$;

$f(x)$ undefined for $x \leq 1$; $\lim_{x \to 1^+} f(x) = -\infty$;

$f'(x) = -2\,\dfrac{x-7}{(x-1)^{5/2}}$; $f''(x) = 3\,\dfrac{x-11}{(x-1)^{7/2}}$;

(10) 2. Find the absolute maximum and minimum for $f(x) = 2\,x^3 - 3\,x^2 - 12\,x$ on the interval $[-3, 3]$.

(10) 3. Use the first derivative test to show $f(x) = 2x^3 - 3x^2 - 12x$ has a local maximum at $x = -1$.

(7) 4. Let $f(x) = x^5 + 10x + 3$ Show $f(x) = 0$ has exactly one real solution.

(8) 5. If $f'(x) \leq g'(x)$ for all $x \leq 0$ and $f(0) = g(0)$, show that $g(x) \leq f(x)$ for all $x \leq 0$.

(10) 6. Find the local extrema for the function $f(x) = xe^{-x^2}$.

(10) 7. Find the points on the hyperbola $y^2 - x^2 = 4$ that are closest to the point (2,0).

(10) 8. Use two iterations of Newton's method, starting with $x = 1$ to find a solution to $x^3 - 2 = 0$

9. Evaluate the following limits

(5) (a) $\lim_{x \to 0} \dfrac{\sin^{-1}(x)}{x}$

(5) (b) $\lim_{x \to 0^+} (\csc x - \cot x)$

(5) (c) $\lim_{x \to \infty} (1 + 2/x)^x$

(10) 10. A piece of wire 10m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How should the wire be cut so the total area is a maximum?

(5) 11. (Extra Credit.) Which theorem is TMITIDC and for what does TMITIDC stand?