M170–004
April 5, 2000

TEST #3 Name _____

This test consists of **4** pages, all different and none intentionally left blank.
Take a minute *right now* to ensure that you have all 4 of these pages.
In order to receive credit for your answers, you must show your work!!

1. (10 points each) First, can you show you know, and can use, the basic formulae? Find the derivative of each of the following functions:

   (a) $f(x) = \sin^{-1}(3x)$

   (b) $g(x) = 2^x \log_{10}(3x + 4)$

2. (10 points) Use implicit differentiation to find $\frac{\mathrm{dy}}{\mathrm{dx}}$ for y defined implicitly by

   $$\sin(xy) = x + y$$

3. (10 points) A tangent to the graph of the parabola $y = x^2$ passes through the point $(-1, -1)$. Where does this tangent touch the parabola? (There are 2 answers.) (Hint: first find an equation of the tangent line to the parabola when $x = a$ and then use this equation and the point $(-1, -1)$ to solve for a).

4. (10 points) Use the definition of $\sinh(x)$ to prove that the derivative of $\sinh(x)$ is $\cosh(x)$.

5. (7 points each) Evaluate the following limits:

(a) $\lim_{x \to 0} \dfrac{\sinh(x) - \sin(x)}{\sin^2(x)}$

(b) $\lim_{x \to 0} (\mathbf{e}^{2x} + 5x)^{3/x}$

(c) $\lim_{x \to 0} \left(\dfrac{\tan x}{x} \right)^{1/x^2}$

6. (10 points) Suppose $f(5) = 2$ and $f'(5) = 4$. If $g = f^{-1}$, find an equation of the tangent to the graph of $g(x)$ at $x = 2$.

7. (10 points) Find linear and quadratic approximations to $f(x) = \ln(x)$ for values of x near 1

8. (6 points) Simplify the following expression

$$\cos(2\arctan(x))$$