

This test consists of 100 points and 5 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

1. Find the derivative of each of the following functions.

(5) (a) $f(x) = x^3 - 7x^2 + 5x - 2$

(5) (b) $f(x) = \sin(3x) \tan(6x)$

(5) (c) $g(x) = \frac{\sqrt{x^2 + 1}}{x - 6}$

(5) (d) $f(x) = \sqrt[4]{x + \sqrt[3]{2x + \sqrt{3x + 4}}}$

Question 1 continued:

(5) (e) $f(x) = \tan^{-1}(x + 2)$

(5) (f) $f(x) = \frac{\cosh(2x)}{\sin^{-1}(x)}$

(10) 2. Use implicit differentiation to find y' for the function y defined implicitly by

$$x^2 + xy + y^2 = \sin(x) \cos(y)$$

- (10) 3. Use logarithmic differentiation to find the derivative of

$$f(x) = (x^2 + 2x + 2)^{x^2+1}$$

4. A calculus student was asked to find an equation of the tangent to the graph of

$$f(x) = \sqrt{x^2 + 9}$$

at the point $(4, 5)$ and gave

$$y - 5 = \frac{x}{\sqrt{x^2 + 9}}(x - 4)$$

as the answer.

- (5) (a) How can you tell at a glance this answer is wrong?

- (5) (b) What is the correct answer?

- (10) 5. Where does the graph of $f(x) = xe^{-x^2}$ have a horizontal tangent?

- (10) 6. Evaluate the following limit. You may find the fact that $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$ to be useful.

$$\lim_{x \rightarrow 0} \frac{\cos(x) - 1}{x}$$

- (10) 7. Use a linear approximation to find a value for $\sqrt[3]{8.2}$

- (10) 8. At noon, boats A and B leave the harbor. Boat A heads northwest at a speed of 15 km/hr and boat B heads east at a speed of 20km/hr. How fast is the distance between the boats changing at 3pm?