

This test consists of 4 pages, none of which is intentionally left blank. Take a few seconds right now to be sure you have all the pages. The point value of each question is to the left of the question number. Show all your work in the space provided. If you run out of room for an answer, continue working on the back of the page. Your answers must be justified by your work.

1. Find and simplify the derivative of each of the following functions:

(5) (a) $f(x) = 3x^2 + 5x - 6$

(5) (b) $f(x) = \ln(\sin(x))$

(5) (c) $g(x) = (x + 2)^{10}(-x + 5)^7$

(5) (d) $h(x) = \frac{x^2}{\sqrt{x^2 + 3}}$

Question 1 continued

(5) (e) $g(x) = x^{\ln(x)}$

(10) 2. Prove that $\cosh(x) + \sinh(x) = e^x$

(10) 3. Find an equation of the line tangent to the graph of $f(x) = x \sec^{-1}(x)$ when $x = -2$

4. The following parts refer to the function $f(x) = \ln(2x)$.
- (5) (a) What is the equation of the line tangent to graph of f at the point $(0.5, 0)$?
- (5) (b) Use a linear approximation to f to approximate $f(0.6)$
- (10) 5. Use implicit differentiation to find y' in terms of x and y if (x, y) satisfies the following
- $$x^5 + xy - y^5 = 1$$
- (10) 6. Find the points on the graph of $f(x) = x^3 + 3x^2 + x - 3$ where the slope of the tangent is 1.

7. When deriving the derivative of $\sin(x)$, we had to derive and use the fact that $\lim_{x \rightarrow 0} \frac{\sin(x)}{x} = 1$. You may find this fact useful in evaluating the following limits. (Calculator answers are not acceptable.)

(7) (a) $\lim_{x \rightarrow 0} \frac{\sin(2x)}{x}$

(8) (b) $\lim_{x \rightarrow 0} \frac{1 - \cos(x)}{x^2}$

- (10) 8. The volume of a sphere of radius r is given by $V = \frac{4}{3}\pi r^3$. If the radius of the sphere is increasing at a rate of 2 cm/min, how fast is the volume increasing when the radius is 6cm.