M170–003
March 10, 1999

TEST #2 Name ______________________________

> This test consists of **4** pages, all different and none intentionally left blank.
> Take a minute *right now* to ensure that you have all 4 of these pages.
> In order to receive credit for your answers, you must show your work!!

1. (6 points each) Find the derivative of each of the following functions.

   (a) $f(x) = 2x^3 + 3x^2 - 5x + 1$

   (b) $f(x) = x\sin(2x)$

   (c) $f(x) = \dfrac{\sqrt{x^2+1}}{x^2-1}$

   (d) $f(x) = (x^2 + x - 1)^3(x^2 + 1)^2$

2. (10 points) Use the definition of the derivative to find $f'(2)$ for $f(x) = \frac{1}{x}$

3. (10 points) Use implicit differentiation to find $\frac{dy}{dx}$ for y defined by $x^2y^2+2xy-\sqrt{y} = 5x$

4. (6 points)Use the intermediate value theorem to show that the equation $x = \cos(x)$ has a solution in the interval $[0, 1]$.

5. (10 points) Find an equation of the line ℓ which is tangent to the graph of $f(x) = \sqrt{x^2+3}$ at the point $(1,2)$

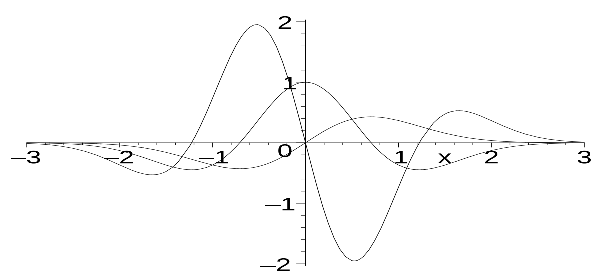
6. The equation $b^2x^2 + a^2y^2 = a^2b^2$ defines an ellipse, centered at the origin.

   (a) (10 points) Find an equation of the line tangent to the ellipse at the point on the ellipse (x_0, y_0)

   (b) (5 points) Find the x and y intercepts of that tangent line.

7. (10 points) A shrinking spherical balloon loses air at the rate of 1 cubic inch per second. At what rate is the radius changing when the radius is (a) 2 inches? (b) 1 inch?

8. (8 points) The following plot shows the graph of a function f, its derivative f' and its second derivative f''. Which is which. Justify your answer.



9. (7 points) Let

$$f(x) = \begin{cases} x^3 - 2\,x & x < 0 \\ x^2 - ax & \text{otherwise} \end{cases}$$

What value of a will make f differentiable at $x = 0$